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BELIEF AND DESIRE: A LOGICAL ANALYSIS

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Abstract

Propositional attitudes directed at facts of the world are cognitive or volitive. Cognitive attitudes contain beliefs and volitive attitudes desires. Beliefs are satisfied whenever they are true and desires whenever they are realized. The main purpose of this paper is to analyze the logical form of such primitive kinds of attitudes and to explicate their conditions of possession and of satisfaction. According to standard epistemic logic human agents are either perfectly rational or totally irrational. I will advocate an intermediate position compatible with philosophy of mind and psychology according to which agents are minimally rational. They can be inconsistent but they never are entirely irrational. In order to account for the imperfect but minimal rationality of human agents I will explicate both *subjective* and *objective possibilities* in the logic of attitudes. For that purpose, I will exploit the resources of a non classical propositional *predicative logic* that distinguishes propositions with the same truth conditions that do not have the same cognitive value. In my approach, the relations of compatibility with the satisfaction of agents' beliefs and desires are also explicated in a finer way so as to avoid current epistemic and volitive paradoxes. I will use both proof- and model-theory in order to formulate my logic of belief and desire. At the end I will enumerate important valid laws governing such attitudes.

As Descartes pointed out in his treatise on *Les passions de l'âme*², beliefs and desires are primitive kinds of attitudes. Every propositional attitude is cognitive or volitive. All cognitive attitudes (e.g. conviction, faith, confidence, knowledge, certainty, presumption, pride, arrogance, surprise, amazement, stupefaction, prevision, anticipation, expectation) contain beliefs. Similarly, all volitive attitudes (e.g. wish, will, intention, ambition, project, hope, aspiration, satisfaction, pleasure, enjoyment, delight, gladness, joy, elation, amusement, fear, regret, sadness, sorrow, grief, remorse, terror) contain desires. So beliefs and desires are essential components of all propositional attitudes. The main purpose of this chapter is to analyze their logical form.

¹ I am grateful to Hugolin Bergier, David Kaplan, Kenneth MacQueen and John Searle for critical remarks on a first draft of this paper.

² The treatise *Les passions de l'âme* is reedited in R. Descartes *Œuvres complètes* La Pléiade Gallimard 1953

Beliefs and desires have *intentionality*: they are *directed at* objects and facts of the world. So these attitudes have *conditions of possession* and of *satisfaction* that are logically related but different. In order that an agent *possesses* a belief or a desire, he must be in a certain mental state. Whoever *possesses* a *belief* represents how things are in the world according to him. Whoever *possesses* a *desire* represents how he prefers things to be in the world. In order that a belief or desire is *satisfied*, represented things must be or turn out to be in the world as the agent represents them. My main objective here is to contribute to the foundations of the logic of attitudes in formulating a general *recursive theory of conditions of possession and of satisfaction of beliefs and desires*, the two primitive kinds of *attitudes*. In my view, more complex psychological modes such as knowledge, certainty, will and intention divide into other components than the *basic traditional categories of cognition and volition*. Complex modes have a *proper way* of believing or desiring, *proper conditions on their propositional content* or *proper preparatory conditions*. I have formulated a recursive definition of all psychological modes and analyzed complex attitudes in another more general paper.³

In the first section I will consider basic problems of the standard logic of propositional attitudes. Like many philosophers of mind, I do not think that human agents are either perfectly rational or totally irrational. On the contrary, I advocate an intermediate position: agents are minimally rational. They can be inconsistent but they never are entirely inconsistent. They do not make all logical inferences but they always make many. We need to consider *subjective* as well as *objective possibilities* in the logic of attitudes and action in order to account for the imperfect but minimal rationality of human agents. For that purpose, I will present in the second section a *predicative logic* that provides a much finer criterion of propositional identity than standard propositional logic. In the third section I will proceed to the analysis of the general categories of cognition and volition. In my approach, the relations of compatibility with the satisfaction of agents' beliefs and desires are explicated in a finer way. Current epistemic and volitive paradoxes are eliminated. I will formulate the ideography and formal semantic of my logic of belief and desire in the fourth section. In the fifth section I will formulate an axiomatic system and in the last section I will enumerate important valid laws governing beliefs and desires.

Section 1 Basic problems of the standard logic of attitudes

³ D. Vanderveken "Fondements de la logique des attitudes" in press in the issue *Language and Thought* of *Manuscrito*. The present paper is based on the theory developed in the first part of that French paper.

Following Carnap⁴, standard propositional logic tends to identify propositions that have the same truth values in the same possible circumstances. However it is clear that strictly equivalent propositions are not the contents of the same attitudes and intentional actions just as they are not the senses of synonymous sentences. We absolutely need a much finer criterion of propositional identity than strict equivalence in logic for the purposes of philosophy of mind, action and language. Maybe Carnap's reduction of Fregean *senses* to *intensions* enables us to define logical truth in the special cases of modal and temporal logics? But it does not work for richer logics dealing with attitudes, actions and illocutions. We need a better logic of sense in order to formulate an adequate theory of meaning, action and thought.

We need first to analyze the logical form of propositions so as to distinguish propositions with the same truth conditions that do not have the same cognitive value. Clearly we do not know *a priori* by virtue of competence the necessary truth of many propositions. We have to learn a lot of essential properties of objects to which we refer. By *essential property* of an object I mean here a property that it really possesses in any possible circumstance.⁵ We discovered in modern times that whales are essentially mammals. So we can ignore necessary truths. We can even be inconsistent and believe necessary falsehoods. (We believed in the past that whales are fishes). However we always remain paraconsistent. As the Greek philosophers already pointed out, we cannot believe that every proposition is true (the sophist's paradox) or false (the skeptic's paradox). Any adequate logic of attitudes and action has to account for such facts. Few necessarily true propositions are pure *tautologies* such as the proposition that whales are whales that we know *a priori* to be true. What is the logical nature of such *tautologies*? My predicative propositional logic gives an answer to that question. It also explains why certain strictly equivalent propositions have a different cognitive value. This solves the first problem of propositional identity.

A second important problem of the standard logic of attitudes is related to the way in which it analyzes the *relations of compatibility with the truth of beliefs and the realization of desires of agents*. According to standard logic, such relations of psychological compatibility are simple modal relations of accessibility existing between agents and moments, on one hand, and

⁴ R. Carnap *Meaning and Necessity* University of Chicago Press 1956.

⁵ See A. Plantinga *The Nature of Necessity* Oxford University Press 1974.

possible circumstances, on the other hand. Thus in Hintikka's epistemic logic⁶, possible circumstances are compatible with the truth of beliefs of agents at each moment of time. To each agent a and moment m there corresponds in each model a unique set $Belief(a,m)$ of possible circumstances that are compatible with the truth of all beliefs that agent a has at moment m . Moreover, according to the standard meaning postulate for belief propositions: an agent a believes a proposition at a moment m when that proposition is true in all possible circumstances belonging to the set $Belief(a,m)$ of circumstances compatible with what that agent then believes.

Given such a formal approach, all human agents are either *perfectly rational* or *totally irrational*. On one hand, they believe all necessarily true propositions. And their beliefs are closed under strict implication. Whoever believes a proposition *eo ipso* believes all propositions that are strictly implied by that proposition. So human agents are perfectly rational when at least one possible circumstance is compatible with what they believe. Otherwise, they are totally irrational. Whoever believes a necessary falsehood *eo ipso* believes all propositions. However, all this is absolutely false according to standard philosophy of mind and empirical psychology. First of all, human agents are *not logically omniscient*. They ignore most logical truths and they do not draw all logical inferences. Moreover even when they are inconsistent, they never are entirely inconsistent. Problems are worse in the case of the logic of desire if we proceed according to the same approach. In that case, to each agent a and moment m there corresponds in each model a unique set $Desire(a,m)$ of possible circumstances that are compatible with the satisfaction of all desires of that agent at that moment. We can make mistakes and wrongly believe necessarily false propositions. However, when we recognize their falsehood, we immediately stop believing them. On the contrary, it is not enough to learn that something is impossible in order to stop desiring it. We keep many desires that we know to be unrealizable. Moreover we never desire everything.

Some have advocated the introduction in epistemic logic of so-called *impossible circumstances* where necessarily false propositions would be true (where, for example, whales would be fishes). However, such a theoretical move is very *ad hoc*. Moreover it is neither necessary nor sufficient. So I prefer to keep only possible circumstances in models while changing the approach. In logic, *possible circumstances* are *objective possibilities* as Belnap says. So objects keep their essential properties (whales are really mammals) and necessarily false

⁶ J. Hintikka "Semantics for Propositional Attitudes" in L. Linsky (ed) *Reference and Modality* Oxford University

propositions remain false in all possible circumstances. However, according to human agents certain necessarily false propositions are true. We did believe that whales are fishes. This is an *epistemic possibility*. So we need to consider *subjective* in addition to *objective possibilities* in logic. Many subjective possibilities are not objective. There is no way to explicate pure subjective possibilities and to define adequately the notion of truth according to an agent within the standard approach. I will enrich the conceptual apparatus of propositional logic so as to analyze the logical form of necessarily false propositions that we can believe and desire. I will also provide a better analysis of the compatibility relation with respect to the satisfaction of attitudes of agents and adopt a finer meaning postulate for defining truth conditions of propositions attributing attitudes.

Section 2 New principles of my predicative propositional logic

My propositional logic is *predicative* in the very general sense that it analyzes the logical form of propositions by taking into account predications that we make in expressing and understanding them.⁷

- In my view, each proposition has a finite *structure of constituents*. It predicates a positive number of *attributes* (properties or relations) of *objects subsumed under concepts*. Each proposition serves to make finitely many predications. As Frege and Russell pointed out, we understand a proposition when we understand which attributes objects must possess in a possible circumstance in order that this proposition be true in that circumstance.

- In addition to taking into account the structure of constituents of propositions, we also need a *better explication of their truth conditions*. We ignore in which possible circumstances most propositions are true because we ignore *real denotations* of most attributes and concepts in many possible circumstances. One can refer to Smith's murderer without knowing who he is. However we can always in principle think of persons who could be that murderer. Sometimes there are even suspects. So in any possible use and interpretation of language, there are a lot of *possible denotation assignments to attributes and concepts* in addition to the standard *real denotation assignment* which associates with each propositional constituent its actual denotation in every possible circumstance. They are functions of the same type. They, for example, associate with individual concepts a unique individual or no individual at all in each possible circumstance. According to a possible denotation assignment, Smith's wife murdered Smith. According to

Press 1962.

another possible assignment, another person is Smith's murderer. According to others, no one murdered Smith. By hypothesis, all possible denotation assignments respect *meaning postulates*. A murderer is not only an individual object; it is a person who has caused death.

We ignore the value of the real denotation assignment for most concepts and attributes in many possible circumstances. But we can in principle think of denotations that they could have. Moreover, when we have in mind certain concepts and attributes only some possible denotation assignments to these senses *are then compatible with* our beliefs. Persons born after Smith's death could not have murdered him. Suppose that according to the chief of police at the beginning of his investigation Smith's murderer is a foreigner. In that case, only possible denotation assignments according to which a foreigner falls under the concept of being Smith's murderer are then compatible with his actual beliefs. So in my approach, possible denotation assignments rather than possible circumstances are compatible with the beliefs of agents in possible circumstances. This is my way to account for subjective possibilities.

- In predicative propositional logic, the *truth definition* is then relative to both possible circumstances and possible denotation assignments. Propositions are *true (or false) in a circumstance according to certain possible denotation assignments* to their constituents. In understanding propositions we in general do not know whether they are true or false. We just know that their truth in a possible circumstance is compatible with certain possible denotation assignments to their attributes and concepts, and incompatible with all others. Thus an elementary proposition predicating an extensional property of an object under a concept (e.g. the proposition that Smith's murderer is tall) is true in a possible circumstance according to a possible denotation assignment if and only if according to that assignment the person who falls under that concept has that property in that circumstance. Otherwise, that proposition is false in that very circumstance according to that assignment. We only know this by virtue of competence. Most propositions have therefore *a lot of possible* in addition to their real *truth conditions*. Suppose that a proposition is true according to a certain possible denotation assignment to its constituents in a certain set of possible circumstances. By definition, that proposition would be true in all and only these possible circumstances if that denotation assignment were the real one.

Of course, in order to be *true in a circumstance* a proposition has to be *true in that circumstance according to the real denotation assignment*. So among all possible truth conditions

⁷ For more information on predicative logic see my collective book *Logic Thought & Action* Springer, 2005.

of a proposition, there are its *real Carnapian truth conditions* which correspond to the set of possible circumstances where it is true according to the real denotation assignment.

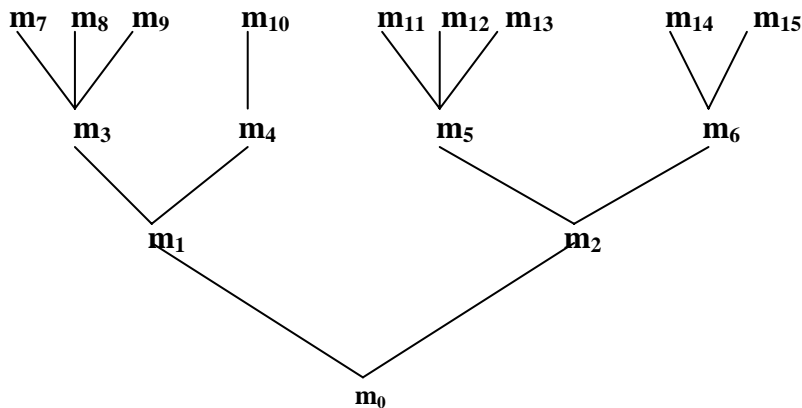
- As one might expect, two propositions *are identical* when they have the same structure of constituents and they are true in the same possible circumstances according to the same possible denotation assignment. Such a finer criterion of propositional identity explains why many strictly equivalent propositions have a different cognitive value. Propositions whose expression requires different acts of predication have a different structure of constituents. So necessarily true propositions about different objects (e.g the propositions that Cicero is Cicero and that Caesar is Caesar) are different. We do not have them in mind at the same moments. My criterion also distinguishes strictly equivalent propositions that we do not understand to be true in the same possible circumstances: these are not true according to the same possible denotation assignments to their constituents.

- My logic accounts for the fact that few necessarily true propositions are *pure tautologies* that are also *a priori* known to be true. In my approach, a proposition is *necessarily true* when it is true in every possible circumstance according to the real denotation assignment. In order to be *tautologically true*, a proposition has to be true in every possible circumstance according to all possible denotation assignments to its constituents. Unlike the proposition that whales are whales, the necessarily true proposition that whales are mammals is not a pure tautology. It is false according to possible denotation assignments according to which whales are fishes. So we can believe that it is false. We now can distinguish in logic subjective and objective possibilities. By definition, a proposition is *subjectively possible* when it is true in a possible circumstance according to at least one possible denotation assignment. In order to be *objectively possible* a proposition has to be true in a possible circumstance according to the real denotation assignment.

Analysis of possible circumstances

In the logic of attitudes and action, the set of possible circumstances is provided with a ramified temporal structure. Human agents are free. Their attitudes and actions are not determined. When they do or think something, they could have done or thought something else. For that reason, one needs a ramified conception of time compatible with indeterminism. In branching time, a *moment* is a complete possible state of the actual world at a certain instant and the *temporal relation* of *anteriority* / *posteriority* between moments is partial rather than linear because of indeterminism. On the one hand, there is a single causal route to the past: each

moment m is preceded by at most one chain of past moments. And all moments are historically connected: any two distinct moments have a common historical ancestor in their past. On the other hand, there are multiple future routes: several incompatible moments might follow upon a given moment. For facts, events or actions occurring at a moment can have incompatible future effects. Consequently, the set of moments of time has the formal structure of a *tree-like frame* of the following form:



A maximal chain h of moments of time is called a *history*. It represents a *possible course of history of our world*. When the history has a first and a last moment, the world has according to it a beginning and an end. A *possible circumstance* is a pair of a moment m and of a history h to which that moment belongs. Thanks to histories logic can analyze important modal notions like settled truth and historic necessity and possibility in addition to temporal notions. Certain propositions are true at a moment according to all histories. Their truth is then *settled at that moment*⁸ no matter how the world continues after it. So are past propositions because the past is unique. Their truth does not depend at all on histories. So are also propositions attributing attitudes to agents. Whoever believes or desires something at a moment then believes or desires that thing no matter what happens later. Contrary to the past, the future is open. The world can continue in various ways after indeterminist moments. Thus the truth of future propositions is not settled at each moment; it depends on which historical continuation h of that moment is under consideration. As Belnap [2001] pointed out, the future proposition that it will be the case that P

(in symbols $WillP$) is true at a moment m according to a history h when the proposition that P is true at a moment m' posterior to m according to that very history. When there are different possible historic continuations of a moment, its actual future continuation is not then determined. However, as Occam pointed out, if the world continues after that moment, it will continue in a unique way. The actual historic continuation of each moment is then unique. Indeterminism does not prevent that unicity. Let h_m be the proper history of moment m . If m is the last moment of a history h , that history is then its proper history h_m . If on the contrary that moment continues, then all moments of its proper history have the same real historic continuation. Among all possible courses of history of this world, one will be its *actual course of history*. It is by hypothesis the proper history of the present actual moment *now*. From now on, I will say that a proposition is *true at a moment* according to a possible denotation assignment when it is then true in the history of that moment according to that assignment.

Section 3 My new approach in the logic of attitudes

The notion of psychological mode is too rich to be taken as a primitive notion. Like Descartes, I consider that the two traditional categories of cognition and volition are essential components of psychological modes. But they divide into other components that we must take into consideration. In my view complex modes of attitudes are obtained from the primitive modes of belief and desire by adding special cognitive or volitive modes, special propositional or preparatory conditions. Here I am only concerned with the primitive attitudes of belief and desire. However, all cognitive modes contain beliefs and all volitive modes desires. So I want to formulate a very general theory of belief and of desire explicating traditional **categories of cognition and volition**. In particular, I advocate, like Searle, a very general explication of volition applying to all kinds of desires directed towards the past (shame), the present (lust) as well as the future (aspiration), even to desires known or believed to be satisfied (pleasure, joy) or unsatisfied (disappointment, regret), including desires directed at past actions that the agent would wish not to have done (remorse).

Beliefs and desires are directed to facts of the world and they have satisfaction conditions. In order that a belief or desire is satisfied, there must be a correspondence between the agent's ideas

⁸ The terminology comes from Belnap [1992].

and things in the world. According to philosophy of mind, *beliefs* have the proper *mind-to-world direction of fit*. Whoever possesses a belief represents how things are in the world. A belief is satisfied when the agent's ideas corresponds to things as they are in the world. On the other hand, *desire* has the opposite *world-to-mind direction of fit*. Volitive attitudes are satisfied only if things in the world are or become as the agent desires them to be. Things in the world must then fit the agent's ideas. Each direction of fit between mind and the world determines which side is at fault in case of dissatisfaction. When a belief turns out to be false, it is the agent who is at fault, not the world. He should have had other ideas about the world. In such a case, the agent easily corrects the situation in changing his ideas. He adopts other beliefs. This is why the *truth predicates* (to be true and to be false) characterize so well *satisfaction* and *dissatisfaction* in the case of belief and other *cognitive* attitudes. However, such truth predicates do not apply to desire and other *volitive* attitudes whose direction of fit goes from things to words. For the world and not the agent is then at fault in the case of dissatisfaction of such attitudes. In the volitive case, the agent in general keeps his ideas and remains dissatisfied. Most often, agents having a *volitive attitude desire the fact represented by the propositional content no matter how that fact turns to be existent in the world*. For that reason, most volitive attitudes are *satisfied* when their propositional content is then true, no matter for which reason. Things are then such that the agent desires them to be, no matter what is the cause of their existence. In the present general explication of cognition and volition, desires like beliefs that agents possess at a moment are satisfied when their propositional content is true at that moment.

I will define the relation of compatibility with the truth of beliefs and the realization of desires of agents in a new way: First of all, in my approach, beliefs, desires and other attitudes of human agents are about objects that they represent under concepts. Each agent has in mind a certain set of attributes and concepts at each moment. (That set is empty when the agent does not exist or is totally unconscious at that moment.) Whoever believes or desires the existence of a fact has in mind all attributes and concepts of a propositional content that represents that fact. Otherwise, he would be unable to determine under which conditions his belief or desire is satisfied. As Wittgenstein and Searle pointed out, an attitude with entirely undetermined satisfaction conditions would be an attitude without real content; it would not be a real attitude. No one can believe or desire to be janissary without understanding the property in question.

Secondly, possible denotation assignments to propositional constituents rather than possible circumstances are compatible with the satisfaction of attitudes of agents. So to each agent a and moment m there corresponds in each model a unique set $Belief(a,m)$ of possible denotation assignments to attributes and concepts that are compatible with the truth of beliefs of that agent at that moment according to that model. By hypothesis, $Belief(a,m)$ is the whole set Val of all possible denotation assignments to senses when the agent a has no sense at all in mind at the moment m . In that case, that agent has then no attitudes. Otherwise, $Belief(a,m)$ is a non empty proper subset of Val . Whenever an agent has in mind propositional constituents, he always respects meaning postulates governing them and *there always are possible denotation assignments compatible with what that agent then believes*. Consider properties that an agent has in mind at a moment. When that agent has no idea at all about the denotation of a property in a circumstance then any possible denotation of a set of individuals under concepts is compatible with what he then believes. Suppose on the contrary that only such and such individuals under concepts could possess that property in that circumstance according to that agent. Then only possible denotation assignments associating with that property in that circumstance a set containing at least one of these individuals under concepts are compatible with what he then believes. And similarly for all other cases.

In my view, an agent a believes a proposition P at a moment m when he or she has then in mind all its constituent senses and that proposition is true at that moment according to all possible denotation assignments belonging to $Belief(a,m)$. We all ignore what will happen later in this world. But we now have a lot of beliefs directed at the future. As Occam pointed out, such beliefs are true when things will be as we believe in *the actual future continuation* of the present moment. Other possible historic continuations do not matter. The same holds for desire. In order that a present desire directed at the future is satisfied, it is not enough that things will be at a posterior moment as the agent now desires. They must be so later in the actual historic continuation of this world.

I will analyze desire according to the same approach. To each agent a and moment m there corresponds in each model a unique non empty set $Desire(a,m)$ of possible denotation assignments to attributes and concepts that are compatible with the satisfaction of all desires of that agent at that moment. There is however an important difference between desire and believe. We can believe, but we cannot desire, facts without believing that they could not exist. For any

desire contains a *preference*. Whoever desires something distinguishes two different ways in which represented things could be in the actual world. In a first preferred way, things are in the world as the agent desires them to be, in a second way, they are not. In the first case, the agent's desire is realized, in the second case, it is unrealized. Thus in order that an agent *a* desires a proposition at a moment *m*, it is not enough that he or she has then in mind all its constituents and that proposition is true at that moment according to all possible denotation assignments belonging to *Desire(a,m)*. That proposition must also not be tautological according to that agent. It must be false at a moment in a history according to possible denotation assignments.

My logic of belief and desire is compatible with philosophy of mind. Given new meaning postulates and its theory of truth, it accounts for the fact that *human agents are not perfectly rational*. We do not have in mind all concepts and attributes. So we ignore logical as well as necessary truths. Our knowledge is limited: we ignore the denotation of certain properties in many circumstances. In that case assignments associating different denotations to these properties in these circumstances are then compatible with our beliefs. We have false beliefs and unsatisfied desires. So many possible denotation assignments compatible with our beliefs and desires do not assign real denotations to attributes that we have in mind. Some of these assignments can even violate essential properties of our objects of reference. In that case we believe necessarily false propositions and desire impossible things. Each human being has a unique mother who gave birth to him or her. However that essential property does not correspond to any meaning postulate. Certain adopted children believe that their adoptive parents are their natural father and mother. One can easily account for such necessarily false beliefs in my analysis. Many traditional epistemic paradoxes are solved.

However *human agents are never totally irrational* according to my approach. On the contrary, they are *minimally rational* in a well determined way. First of all, they cannot believe or desire everything since some possible denotation assignments are always compatible with the satisfaction of their beliefs and desires. Moreover, they cannot possess certain beliefs and desires without possessing others. For all possible denotation assignments compatible with their beliefs and desires and consequently the contents of such attitudes have to respect meaning postulates. So human agents are in a sense *minimally omniscient* as regards logical truth; they cannot have in mind a pure tautology without knowing for certain that it is necessarily true. Represented things could not be in another way according to them. Similarly, so called *pure contradictions* (that is to

say negations of tautologies) are false in every possible circumstance according to any possible denotation assignment. So we can neither believe nor desire contradictory things. Things could never be in a contradictory way according to us. Actual logicians still hope that arithmetic is complete (a necessarily false proposition if Gödel's proof is right). But no one could believe and desire both the completeness and the incompleteness of arithmetic (a pure contradiction). Sometimes we desire something (to be somewhere at a moment) for one reason and another incompatible thing (to be elsewhere at the same moment) for another reason. But when the logical form of such attitudes is fully analyzed, they are not desires with a contradictory content. Although agents believe all tautological propositions that they have in mind, they cannot desire the truth of such tautologies. As I said earlier, in order to desire a fact one must believe that it could not be the case. One can desire to drink; one can also desire not to drink. But one cannot desire to drink or not drink.

Incidentally, there is in predicative logic a new *strong* propositional *implication* much finer than Lewis' strict implication that is important for the analysis of psychological commitment. Let us say that a proposition P *strongly implies* another Q when firstly it contains the same or more predications than Q and it is tautologically true that if P then Q. Strong implication is finite, paraconsistent, decidable and *a priori known*.⁹ As one might expect, any agent who believes a proposition also believes any proposition that is strongly implied by it. For that agent cannot have in mind the first proposition without having in mind the second and without understanding that the first cannot be true unless the second is. Unlike belief, desire is not closed under strong implication. Whoever desires to drink does not desire to drink or not drink.

I have just explained my way to analyze conditions of possession and satisfaction of beliefs and desires. In my view, the two traditional categories of cognition and volition correspond to the two families of compatibility relations $Belief_m^a$ and $Desire_m^a$ that exist between agents and moments, on one hand, and possible denotation assignments, on the other hand. We of course have real attitudes about objects at certain moments in this actual world. But we could have had other attitudes about the same or even about other objects. Often other agents attribute to us attitudes that we do not have. So compatibility relations $Belief_m^a$ and $Desire_m^a$ are moreover

⁹ See chapter 10 "Propositional Identity, Truth According to Predication and Strong Implication" in my book *Logic, Thought and Action* Springer 2005.

relative to possible denotation assignments in models. Agents' attitudes can differ according to different possible denotation assignments. First of all, each agent could have many concepts and attributes in mind. So in each model, every agent a has in mind at each moment m a *certain set* $val(a,m)$ of *propositional constituents* according to any possible denotation assignment val . When this set is not empty, the agent then possesses beliefs and desires according to the assignment in question.

By hypothesis, $Belief_m^a(val)$ is the non empty set of all possible denotation assignment that are compatible with the truth of beliefs that agent a has at moment m according to denotation assignment val . Similarly, $Desire_m^a(val)$ is the non empty set of all possible denotation assignments that are compatible with the satisfaction of desires that agent a has at moment m according to assignment val . Of course, $Belief_m^a(val)$ and $Desire_m^a(val)$ are the whole set Val of all possible denotation assignment when there is no propositional constituent in the set $val(a,m)$. In my approach, an agent a *believes* or *desires* a proposition at a moment m (no matter what is the history) according to a possible denotation assignment val when firstly, he then has in mind all its concepts and attributes (they belong to the set $val(a,m)$) and secondly that proposition is then true at that moment according to all possible assignments of the set $Belief_m^a(val)$ or $Desire_m^a(val)$. In the case of desire, the desired proposition must moreover be false in a circumstance according to the agent at the moment m .

As in modal logic, formal properties of psychological compatibility relations that correspond to attitudes depend on their nature. Whoever has a belief believes that he has that belief. So the relation $Belief_m^a(val)$ is transitive in each model. On the contrary, we often feel desires that we would wish not to feel. The relation $Desire_m^a$ is then not transitive. Some of our beliefs are false; many of our desires are unsatisfied. The compatibility relations $Belief_m^a(val)$ and $Desire_m^a(val)$ are also not reflexive. They are moreover not symmetric.

Section 4 Formal semantics for belief and desire

The **vocabulary of the ideal object language** \mathcal{L} of my elementary logic of attitudes contains a series of *individual constants* naming individual objects or agents, for each positive natural number n , a series of *predicates of degree* n expressing relations of degree n and the

syncategorematic expressions: \wedge , \neg , *Settled*, \square , *Actually*, *Tautological*, \geq , *Bel*, *Des*, *Was*, *Will*, (and).

Rules of formation of the ideography \mathcal{L}

If R_n is a predicate of degree n and $t_1, \dots, \text{ and } t_n$ are n individual terms, then $(R_n t_1 \dots t_n)$ is a propositional formula. If a an individual term and A_p and B_p are propositional formulas, then $\neg A_p$, $\square A_p$, *Tautological* A_p , *Bela* A_p , *Desa* A_p , $(A_p \geq B_p)$ and $(A_p \wedge B_p)$ and are new complex propositional formulas. $(R_n t_1 \dots t_n)$ expresses the *elementary proposition* which predicates the attribute expressed by R_n of the n individuals under concepts expressed by t_1, \dots, t_n in that order. $\neg A_p$ expresses the *negation* of the proposition that A_p . *Settled* A_p expresses the *modal proposition* that it is settled that A_p . $\square A_p$ expresses the *modal proposition* that it is then necessary that A_p . *Actually* A_p expresses the *proposition* that it is actually true that A_p .¹⁰ *Was* A_p and *Will* A_p respectively express the temporal *proposition* that it was and that it will be the case that A_p . *Bela* A_p and *Desa* A_p respectively express the *proposition* that the individual named by a believes and desires that A_p . $(A_p \wedge B_p)$ expresses the *conjunction* of the two propositions that A_p and that B_p . $(A_p \supseteq B_p)$ means that all predications made in expressing the proposition that B_p are predications made in expressing the proposition that A_p . Finally, *Tautological* A_p means that the proposition expressed by A_p is tautological.

Rules of abbreviation

I will use standard rules of abbreviation for elimination of parentheses and truth or modal connectives of *disjunction* \vee , *material implication* \Rightarrow , *material equivalence* \Leftrightarrow , *possibility* and *strict implication* $\text{---}\in$. Here are new rules:

Same structure of constituents: $A_p \equiv B_p =_{\text{df}} (A_p \geq B_p) \wedge (B_p \geq A_p)$

Identical individual concepts: $\wedge t_1 = \wedge t_2 =_{\text{df}} (R_1 t_1) \geq (R_1 t_2)$ where R_1 is the first unary predicate

Identical attributes: $\wedge R_n = \wedge R'_n =_{\text{df}} (R_n t_1 \dots t_n) \geq (R'_n t_1 \dots t_n)$

Always $A =_{\text{def}} (\neg \text{Was} \neg A) \wedge A \wedge \neg \text{Will} \neg A$

Historical possibility: $\diamond A =_{\text{def}} \neg \square \neg A$

¹⁰ The proposition *Actually* A_p is true at a moment according to a history when it is true at that moment in its proper history.

Universal Necessity $\blacksquare A =_{\text{def}} \text{Always} \Box A \wedge \Box \text{Always} A$

*Analytic implication*¹¹: $(A_p \rightarrow B_p) =_{\text{def}} (A_p \geq B_p) \wedge (A_p \text{---} \in B_p)$

Strong implication: $A_p \mapsto B_p =_{\text{def}} (A_p \geq B_p) \wedge \text{Tautological}(A_p \Rightarrow B_p)$

Propositional identity: $A_p = B_p =_{\text{def}} (A_p \mapsto B_p) \wedge B_p \mapsto A_p$

Strong psychological commitment $\text{Bel}A_p \blacktriangleright \text{Bel}B_p =_{\text{def}} \text{Tautological}(\text{Bela}A_p \Rightarrow \text{Bela}B_p)$

Weak psychological commitment $\text{Bel}A_p \triangleright \text{Bel}B_p =_{\text{def}} \text{Tautological}(\text{Bela}A_p \Rightarrow \neg \text{Bela}\neg B_p)$

And similarly for *Des*.

Definition of a model structure

A *standard model* \mathcal{M} for \mathcal{L} is a structure $\langle \text{Moments}, \text{Individuals}, \text{Agents}, \text{Concepts}, \text{Attributes}, \text{Val}, \text{Predications}, \text{Belief}, \text{Desire}, *, \otimes, \parallel \rangle$ that satisfies the following conditions:

- The set *Moments* is a set of *moments of time*. It is partially ordered by a temporal relation \leq as in ramified temporal logic. $m_1 < m_2$ means that moment m_1 is *anterior to* moment m_2 . By definition, $<$ is subject to *historical connection* and *no downward branching*. Any two distinct moments have a common historical ancestor. Moreover, the past is unique. A maximal chain h of moments is called a *history*. It represents a possible course of history of the world. The set *Circumstances* of all *possible circumstances* contains all pairs m/h where m is a moment belonging to the history h . Among all histories to which belongs a moment m there is one h_m representing how the world would continue after that moment. If $m' \in h_m$, $h_{m'} = h_m$. The set *Instants*, whose elements ι, ι', \dots are called *instants*, is a partition of the set *Time* which satisfies *unique intersection* and *order preservation*. So to any instant ι and history h there corresponds a unique moment $m(\iota, h)$ belonging to both ι and h . And $m(\iota, h) \leq m(\iota', h)$ when $m(\iota, h') \leq m(\iota', h')$. Two moments of time m and m' are *coinstantaneous* (in symbols: $m \cong m'$) when they belong to the same instant. Coinstantaneous moments m and m' represent two complete possible states of the world in which things could be at a certain instant.

- *Individuals* is a set of possible *individual objects*. For each moment m , Individuals_m is the set of individuals existing at that moment. *Agents* is a subset of *Individuals* containing *persons*.

¹¹ See W.T. Parry, "Ein Axiomensystem fuer eine neue Art von Implikation (analytische Implikation)", *Ergebnisse eines Mathematisches Colloquiums*, Volume 4, 1933 and "Comparison of entailment theories", *The Journal of Symbolic Logic*, Volume 37, 1972 as well as K. Fine, "Analytic implication", in *Notre Dame Journal of Formal Logic*, Volume 27, number 2, April 1986.

- *Concepts* is the set of *individual concepts* and *Attributes* is the set of *attributes* of individuals considered in model \mathcal{M} . For each natural number n , $Attributes(n)$ is the subset of *Attributes* containing all *attributes* of degree n .

- The set *Val* is a proper subset of $((Concepts \times Circumstances) \rightarrow (Individuals \cup \{\emptyset\})) \cup \bigcup_n ((Attributes(n) \times Circumstances) \rightarrow \mathcal{P}(Concepts^n))$. *Val* contains all possible *denotation assignments* of the model \mathcal{M} . Such assignments are also called possible *valuations of constituents*. For any possible circumstance m/h , $val(c_e, m/h) \in Individuals$ when individual concept c_e has a denotation in the circumstance m/h according to assignment val . Otherwise $val(c_e, m/h) = \emptyset$. For any attribute R_n of degree n , $val(R_n, m/h) \in \mathcal{P}(Concepts^n)$. The set *Val* contains a *real valuation* $val_{\mathcal{M}}$ which assigns to concepts and attributes their *actual denotation* in each possible circumstance according to the model \mathcal{M} . Moreover, there corresponds to each agent a , moment m and assignment val a particular set $val(a, m)$ containing all propositional constituents that the agent a has in mind at that moment according to that assignment.

- *Belief* is a function from $Agents \times Moments \times Val$ into $\mathcal{P}(Val)$ that associates with any agent a , moment m and valuation val , a non-empty set $Belief_m^a(val) \subseteq Val$ containing all possible denotation assignments which are compatible with the beliefs that agent a has at the moment m according to that valuation. The relation of epistemic compatibility corresponding to $Belief_m^a$ is transitive. Moreover, $val(a, m) \subseteq val'(a, m)$ when $val' \in Belief_m^a(val)$.

- *Desire* is a function from $Agents \times Moments \times Val$ into $\mathcal{P}(Val)$ that associates with any agent a , moment m and valuation val , a non empty set $Desire_m^a(val) \subseteq Val$ containing all possible denotation assignments which are compatible with the desires that agent a has at moment m according to that valuation.

As one can expect, $Val = Belief_m^a(val) = Desire_m^a(val)$ when $val(a, m) = \emptyset$.

- The set *Predications* is a subset of $\mathcal{P}(Attributes \cup Concepts)$ containing all sets of propositional constituents with which *predications* that can be made in the language \mathcal{L} . Each member of that set is a set of the form $\{R_n, c^1, \dots, c^k\}$ containing a single attribute R_n of degree n and a number k of individual concepts $k \leq n$ with $k \neq 0$ when n is positive. The power set $\mathcal{P}Predications$ is closed

under union \cup , a modal unary operation $*$ and, for each agent concept a , a unary epistemic operation \otimes_a of the following form: For any Γ, Γ_1 and $\Gamma_2 \in \mathcal{PPredications}$, $\Gamma \subseteq *\Gamma$ and $*\Gamma \subseteq \otimes_a \Gamma$. Moreover, $*(\Gamma_1 \cup \Gamma_2) = *\Gamma_1 \cup *\Gamma_2$ and $**\Gamma = *\Gamma$. Similarly, $\otimes_a(\Gamma_1 \cup \Gamma_2) = \otimes_a \Gamma_1 \cup \otimes_a \Gamma_2$ and $\otimes_a \otimes_a \Gamma = \otimes_a \Gamma$. By definition, when $\cup \Gamma \subseteq \text{val}(a,m)$, $\otimes_a \Gamma \subseteq \text{val}(a,m)$.¹²

- Finally, $\| \cdot \|$ is a function that associates with each expression of \mathcal{L} the sense of that expression in the possible interpretation \mathcal{M} . $\|A\|$ satisfies the following clauses:

- For any individual constant of \mathcal{L} $\|c_\alpha\| \in \text{Concepts}$.

- For any predicate R_n of degree n , $\|R_n\| \in \text{Attributes}(n)$.

- For any propositional formula A_p , $\|A_p\|$ belongs to a subset of $\mathcal{PPredications} \times (\text{Circumstances} \rightarrow \mathcal{PVal})$. Remember that each proposition P has two essential features: the set $id_1 P$ of all its predications and the set $id_2 P$ of all possible denotation assignments according to which it is true. Consequently:

- $id_1 \|[(R_n c_1, \dots, c_n)]\| = \{\{\|R_n\|, \|t_1\|, \dots, \|t_n\|\}\}$ and $id_2 \|[(R_n c_1, \dots, c_n)]\|(m/h) = \{f \in \text{Val} / \langle \|c_1\|, \dots, \|c_n\| \rangle \in f(\|R_n\|, (m/h))\}$.

- $id_1 \|\neg A_p\| = id_1 \|A_p\|$ and $id_2 \|\neg A_p\|(m/h) = \text{Val} - id_2 \|A_p\|(m/h)$.

- $id_1 \|\Box A_p\| = * id_1 \|A_p\|$ and $id_2 \|\Box A_p\|(m/h) = \bigcap_{m' \cong m} \bigcap_{m' \in h'} id_2 \|A_p\|(m'/h')$.

- $id_1 \|Will A_p\| = * id_1 \|A_p\|$ and $id_2 \|Will A_p\|(m/h) = \bigcup_{m' > m} id_2 \|A_p\|(m'/h)$.

- $id_1 \|Was A_p\| = * id_1 \|A_p\|$ and $id_2 \|Was A_p\|(m/h) = \bigcup_{m' < m} id_2 \|A_p\|(m'/h)$.

- $id_1 \|Settled A_p\| = * id_1 \|A_p\|$ and $id_2 \|Settled A_p\|(m/h) = \bigcap_{m \in h} id_2 \|A_p\|(m/h)$.

- $id_1 \|Actually A_p\| = * id_1 \|A_p\|$ and $id_2 \|Actually A_p\|(m/h) = id_2 \|A_p\|(m/h_m)$.

- $id_1 \|Tautological A_p\| = * id_1 \|A_p\|$ and $id_2 \|Tautological A_p\|(m/h) = \bigcap_{m' / h'} id_2 \|A_p\|(m'/h')$.

- $id_1 (\|B_p \wedge C_p\|) = id_1 (\|B_p\|) \cup id_1 (\|C_p\|)$; $id_2 \|B_p \wedge C_p\|(m/h) = id_2 \|B_p\|(m/h) \cap id_2 \|C_p\|(m/h)$.

¹² As one can expect, each agent who has attitudes has attitudes about himself.

- $id_1(\|B_p \geq C_p\|) = *(id_1(\|B_p\|) \cup id_1(\|C_p\|))$ and $id_2\|B_p \geq C_p\|(m/h) = Val$ when $id_1\|B_p\| \subseteq id_1\|C_p\|$. Otherwise, $id_2\|B_p \geq C_p\|(m/h) = \emptyset$.

- $id_1\|BelaB_p\| = \otimes_{\|a\|} id_1\|B_p\|$ and $id_2\|BelaB_p\|(m/h) = \{val / \cup\|B_p\| \subseteq val(a,m) \text{ and } ((\|A_\mu\|(val) \cap Belief_m^a(val)) \subseteq id_2\|B_p\|(m/h_m) \text{ where } a = val\|a\|(m,h)\}$. And similarly for $\|DesaB_p\|_c^\sigma$ with the additional condition that for some m and h , $id_2\|B_p\|(m/h) \neq Val$.

Definition of truth and validity

A propositional formula A_p of \mathcal{L} is *true* in a possible circumstance m/h according to a standard model \mathcal{M} if and only $\|A_p\|$ is true in m/h according to $val\mathcal{M}$. The formula A_p is *valid* (in symbols: $\models A_p$) when it is true in all possible circumstances according to all standard models.

Section 4 An axiomatic system

I conjecture that one can prove all and only valid formula of \mathcal{L} containing syncategorematic symbols $\wedge, \neg, \square, Tautological, \geq, Bel, Des, (\text{ and })$ in the following axiomatic system \mathcal{S} :

Axioms

The axioms of \mathcal{S} are all the instances in that sub-language of \mathcal{L} of classical axiom schemas of truth functional logic and S5 modal logic as well as instances of the following new schemas:

Axiom schemas for tautologies

$$(T1) \text{ Tautological } A_p \Rightarrow \square A_p$$

$$(T2) \text{ Tautological } A_p \Rightarrow \text{ Tautological Tautological } A_p$$

$$(T3) \neg \text{ Tautological } A_p \Rightarrow \text{ Tautological } \neg \text{ Tautological } A_p$$

$$(T4) \text{ Tautological } A_p \Rightarrow (\text{ Tautological } (A_p \Rightarrow B_p) \Rightarrow \text{ Tautological } B_p)$$

$$(T5) (A_p \geq B_p) \Rightarrow \text{ Tautological } (A_p \geq B_p)$$

$$(T6) \neg (A_p \geq B_p) \Rightarrow \text{ Tautological } \neg (A_p \geq B_p)$$

Axiom schemas for propositional identity

$$(I1) A_p = A_p$$

(I2) $(A_p = B_p) \Rightarrow (C \Rightarrow C^*)$ where C^* and C are propositional formulas which differ at most by the fact that an occurrence of B_p in C^* replaces an occurrence of A_p in C .

$$(I3) (A_p = B_p) \Rightarrow \textit{Tautological} (A_p = B_p)$$

$$(I4) \neg(A_p = B_p) \Rightarrow \textit{Tautological} \neg(A_p = B_p)$$

Axiom schemas for belief

$$(B1) (BelaA_p \wedge BelaB_p) \Rightarrow Bela(A_p \wedge B_p)$$

$$(B2) \textit{Tautological}A_p \Rightarrow \neg Bela\neg A_p$$

$$(B3) \textit{Tautological}A_p \Rightarrow (BelaA_p \Rightarrow Bela\textit{Tautological}A_p)$$

$$(B4) BelaA_p \Rightarrow ((A_p \mapsto B_p) \Rightarrow (BelaB_p))$$

$$(B5) BelaA_p \Leftrightarrow (BelaBelaA_p)$$

$$(B6) BelaA_p \Rightarrow Bela\Diamond A_p$$

Axiom schemas for desire

$$(D1) (DesaA_p \wedge DesaB_p) \Rightarrow Desa(A_p \wedge B_p)$$

$$(D2) \textit{Tautological}A_p \Rightarrow \neg(DesaA_p \vee Desa\neg A_p)$$

$$(D3) DesaA_p \Rightarrow (((A_p \mapsto B_p) \wedge \neg\textit{Tautological}A_p) \Rightarrow (DesaB_p))$$

$$(D4) DesaA_p \Rightarrow Bela\neg\textit{Tautological}A_p$$

$$(D5) DesaA_p \Rightarrow Desa\Diamond A_p \wedge Bela\neg\blacksquare A_p$$

Axiom schemas for propositional composition

$$(C0) ((R_n t_1, \dots, t_n) > A_p) \Rightarrow (A_p = (R_n t_1, \dots, t_n))$$

$$(C1) A_p \geq A_p$$

$$(C2) (A_p \geq B_p) \Rightarrow ((B_p \geq C_p) \Rightarrow (A_p \geq C_p))$$

$$(C3) (A_p \wedge B_p) \geq A_p$$

$$(C4) (A_p \wedge B_p) \geq B_p$$

$$(C5) ((C_p \geq A_p) \wedge (C_p \geq B_p)) \Rightarrow C_p \geq (A_p \wedge B_p)$$

$$(C6) A_p \equiv \neg\neg A_p$$

(C7) $\Box A_p \equiv \text{Tautological}A_p$

(C8) $(A_p \geq B_p) \equiv \Box(A_p \wedge B_p)$

(C9) $\Box A_p \geq A_p$

(C10) $BelaA_p \geq \Box A_p$ And similarly for *Desa*.

(C11) $\Box \neg A_p \equiv \Box A_p$ And similarly for *Bela* and *Desa*. (C14)

(C12) $\Box(A_p \wedge B_p) \equiv (\Box A_p \wedge \Box B_p)$ And similarly for *Bela* and *Desa*. (C16)

(C13) $\Box \Box A_p \equiv \Box A_p$ And similarly for *Bela* and *Desa*. C18)

Rules of inference

The two rules of inference of my axiomatic system are:

The *rule of Modus Ponens*: (MP) From sentences of the form A and $(A \Rightarrow B)$ infer B .

The *tautologization rule*: (RT) From a theorem A infer *Tautological*A .

Section 5 Valid laws

Here are important valid laws of my logic of belief and desire.¹³

Laws of structure of constituents A proposition has all elementary propositions of its arguments. $\models A_p \geq (R_{nt_1, \dots, c_n})$ when (R_{nt_1, \dots, c_n}) occurs in A_p . Modal and epistemic propositions have in general more elementary propositions than their arguments. Thus $\not\models A_p \geq \blacksquare A_p$ and $\not\models \blacksquare A_p \geq BelaA_p$.

Laws for tautologyhood Tautologyhood is stronger than necessary truth and contradiction stronger than necessary falsehood. $\models (\text{Tautological}A_p) \Rightarrow \blacksquare A_p$. But $\not\models \blacksquare A_p \Rightarrow \text{Tautological}A_p$

Some tautologies are modal and epistemic. Thus $\models \text{Tautological} (\blacksquare A_p \Rightarrow A_p)$.

Agents are minimally rather than perfectly rational.

They do not believe all necessary truths and they can believe and desire necessarily false propositions. Thus $\not\models \blacksquare A_p \Rightarrow BelaA_p$ and $\not\models \neg \blacklozenge A_p \Rightarrow Bela \neg A_p$. However they are minimally

consistent: they cannot believe that a tautology is false or that a contradiction is true.

$\models \text{Tautological} \neg A_p \Rightarrow \neg \text{Bela} A_p$ Moreover they neither desire tautologies nor contradictions (axiom D3). Now in order to believe or desire a proposition an agent must have in mind its attributes and concepts. Unlike God, human agents do not have in mind all propositional constituents. Consequently they do not know or even believe all tautologies. $\not\models \text{Tautological} A_p \Rightarrow \text{Bela} A_p$. The limits of their language impose limits to their thoughts. However whenever they express a tautology and a contradiction, they know just by apprehending their logical form that the first is necessarily true and the second necessarily false (axiom B3).

Laws for tautological implication

Tautological implication is much finer than strict implication. $\models \text{Tautological} (A_p \Rightarrow B_p) \Rightarrow (A_p \text{---} \in B_p)$ But $\not\models (A_p \text{---} \in B_p) \Rightarrow \text{Tautological} (A_p \Rightarrow B_p)$. Necessarily true propositions are strictly implied by others. $\models \blacksquare A_p \Rightarrow (B_p \text{---} \in A_p)$. But only tautologies can tautologically imply other tautologies. $\models ((\text{Tautological} B_p) \wedge \text{Tautological} (A_p \Rightarrow B_p)) \Rightarrow \text{Tautological} A_p$. So $\not\models \blacksquare A_p \Rightarrow \text{Tautological} (B_p \Rightarrow A_p)$. Similarly, necessarily false propositions strictly imply all other propositions. $\models \blacksquare \neg A_p \Rightarrow (A_p \text{---} \in B_p)$. But only contradictions can tautologically imply contradictions. So $\not\models \blacksquare \neg A_p \Rightarrow \text{Tautological} (A_p \Rightarrow B_p)$.

Beliefs are not closed under tautological implication. $\not\models (\text{Tautological} (A_p \Rightarrow B_p)) \Rightarrow (\text{Bela} A_p \Rightarrow \text{Bela} B_p)$ Because $\not\models (\text{Tautological} (A_p \Rightarrow B_p)) \Rightarrow (A_p \geq B_p)$. However whoever believes a proposition cannot believe the negation of a proposition that the first tautologically implies. For the conjunction of both is a contradiction. This is why tautological implication generates *weak psychological and illocutionary commitment*. Any assertion that P *weakly commits* the agent to asserting any proposition Q that P tautologically imply according to illocutionary logic.¹⁴

Similarly, $\models \text{Tautological} (A_p \Rightarrow B_p) \Rightarrow (\text{Bela} A_p \text{---} \in \neg \text{Bela} \neg B_p)$ and $\models \text{Tautological} (A_p \Rightarrow B_p) \Rightarrow (\text{Desa} A_p \text{---} \in \neg \text{Desa} \neg B_p)$

¹³ Some of these laws are stated in my paper “Truth, Belief and Certainty in Epistemic Logic” in E. Maier *et al Proceedings of Sinn und Bedeutung 9* NCS Nijmegen 2005

¹⁴ See “Success, Satisfaction and Truth in the Logic of Speech Acts and Formal Semantics” [2004]

Laws for strong implication

Strong implication is a stronger kind of implication than strict, tautological and analytic implications. It requires the same or a richer structure of constituents in addition to tautological implication. There are two reasons why a proposition can fail to strongly imply another. Firstly, that other proposition requires new predications. $\models \neg(A_p \geq B_p) \Rightarrow \neg(A_p \mapsto B_p)$. In that case, one can think the first proposition without thinking the second. Secondly, the first proposition does not tautologically imply the other. In that case one can ignore that it implies the other.

Strong implication is then finer than analytic implication. $\not\models (A_p \rightarrow B_p) \Rightarrow (A_p \mapsto B_p)$ In particular $\not\models (A_p \rightarrow B_p) \Rightarrow BelaA_p \Rightarrow Bela B_p$. Unlike strict and tautological implications, strong implication is anti-symmetric. Consequently, $\models A_p \mapsto B_p \Leftrightarrow ((A_p \wedge B_p) = A_p)$

Strong implication is *decidable*. For $\models A_p \geq B_p$ when all propositional constants which occur in B_p also occur in A_p . And $\models Tautological (A_p \Rightarrow B_p)$ when any semantic tableau of S5 modal logic for $(A_p \Rightarrow B_p)$ closes.

Moreover, strong implication is *finite*: every proposition only strongly implies a finite number of others. For it contains a finite number of elementary propositions. In particular, a proposition only strongly implies tautologies having its elementary propositions. $\models Tautological B_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p \geq B_p)$. A contradiction only strongly propositions having its elementary propositions.

$$\models Tautological \neg A_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p \geq B_p)$$

For all these reasons, strong implication is *known a priori*. $\models (A_p \mapsto B_p) \Rightarrow (BelaA_p \Rightarrow Bela(A_p \Rightarrow B_p))$. However \mapsto does not obey the rule of *Modus Tollens*. $\not\models (A_p \mapsto B_p) \Rightarrow (\neg B_p \mapsto \neg A_p)$.

For $\not\models (A_p \mapsto B_p) \Rightarrow (B_p \geq A_p)$. So $\not\models (A_p \mapsto B_p) \Rightarrow (Bela\neg B_p \Rightarrow Bela\neg A_p)$

Natural deduction

Valid laws of inference of natural deduction generate strong implication when their premises contain all propositional constants of their conclusion. Here are some laws:

The law of introduction of belief: $\models A_p \mapsto B_p \Rightarrow BelaA_p \mapsto BelaB_p$

The law of introduction of desire: $\models ((A_p \mapsto B_p) \wedge \neg \text{Tautological}A_p) \Rightarrow \text{Desa}A_p \mapsto \text{Desa}B_p$

The law of elimination of conjunction: $\models (A_p \wedge B_p) \mapsto A_p$ and $\models (A_p \wedge B_p) \mapsto B_p$

The law of elimination of disjunction: $\models ((A_p \mapsto C_p) \wedge (B_p \mapsto C_p)) \Rightarrow (A_p \vee B_p) \mapsto C_p$

Failure of the law of introduction of disjunction: $\not\models A_p \mapsto (A_p \vee B_p)$.

Consequently, strong implication is stronger than *entailment* which obeys the law of introduction of disjunction. Clearly $\not\models \text{Bela}A_p \mapsto \text{Bela}(A_p \vee B_p)$. Similarly, $\not\models \text{Desa}A_p \mapsto \text{Desa}(A_p \vee B_p)$.

The law of introduction of negation: $\models A_p \mapsto O_t \Rightarrow (A_p \mapsto \neg A_p)$ where O_t is any contradiction.

Failure of the law of elimination of negation: $\not\models (A_p \wedge \neg A_p) \mapsto B_p$

Agents can have relatively inconsistent beliefs and desires. $\not\models (A_p \text{ ---} \neg B_p) \Rightarrow \neg \blacklozenge \text{Bela}(A_p \wedge B_p)$ Similarly, $\not\models (A_p \text{ ---} \neg B_p) \Rightarrow \neg \blacklozenge \text{Desa}(A_p \wedge B_p)$

They are *paraconsistent*. $\not\models (A_p \text{ ---} \neg B_p) \Rightarrow (\text{Bela}(A_p \wedge B_p) \Rightarrow \text{Bela}C_p)$

But they always respect the principle of non contradiction. $\models \neg \blacklozenge \text{Bela}(A_p \wedge \neg A_p)$

The law of elimination of material implication: $\models (A_p \wedge (A_p \Rightarrow B_p)) \mapsto B_p$

The law of elimination of necessity: $\models \blacksquare A_p \mapsto A_p$

The law of elimination of possibility: $\models \blacklozenge A_p \mapsto B_p \Rightarrow A_p \mapsto B_p$

Laws of propositional identity

All the classical *Boolean laws of idempotence, commutativity, associativity and distributivity* are valid laws of propositional identity: So $\models \text{Bela}A_p = \text{Bela}(A_p \wedge A_p)$; $\models \text{Bela}(A_p \wedge B_p) = \text{Bela}(B_p \wedge A_p)$; $\models \text{Bela}\neg(A_p \vee B_p) = \text{Bela}(\neg A_p \wedge \neg B_p)$; $\models \text{Bela}(A_p \wedge (B_p \vee C_p)) = \text{Bela}((A_p \wedge B_p) \vee (A_p \wedge C_p))$ and $\models \text{Bela} \blacksquare(A_p \wedge B_p) = \text{Bela}(\blacksquare A_p \wedge \blacksquare B_p)$.

The classical laws of *reduction* are also valid: $\models \neg\neg A_p = A_p$ and $\models \text{Bela}\text{Bela}A_p = \text{Bela}A_p$

However, $\not\models \text{Desa}A_p \Rightarrow \text{Desa}\text{Desa}A_p$. Unlike hyperintensional logic, predicative logic does not

require that identical propositions be *intensionally isomorphic*.¹⁵ First of all, as I said earlier, the order of predication does not always affect truth conditions. Similarly, the order and number of applications of propositional operations does not always affect the logical form. Clearly, $\models Bela(A_p \Leftrightarrow B_p) = Bela(B_p \Leftrightarrow A_p)$ Intensional isomorphism is too strong a criterion of propositional identity.

However, propositional identity requires more than the *co-entailment* advocated in the logic of relevance. As M. Dunn points out, it is unfortunate that A_p and $(A_p \wedge (A_p \vee B_p))$ co-entail each other.¹⁶ For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new sense. $\not\models A_p \mapsto (A_p \wedge (A_p \vee B_p))$. Consequently $\not\models Bela A_p \mapsto Bela (A_p \wedge (A_p \vee B_p))$.¹⁷

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¹⁵ See Max J. Cresswell, "Hyperintensional Logic". *Studia Logica* [1975].

¹⁶ See his philosophical ruminations in Anderson *et al* [1992].

¹⁷ A general predicative logic of propositions dealing with generalization, logical and historic modalities, ramified time, attitudes and action is fully developed in my next book *Logic Truth & Thought*.

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