Chapter 10

PROPOSITIONAL IDENTITY, TRUTH ACCORDING TO PREDICATION AND STRONG IMPLICATION*

With a Predicative Formulation of Modal Logic

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Abstract

In contemporary philosophy of language, mind and action, propositions are not only *senses of sentences* with truth conditions but also *contents of conceptual thoughts* like illocutionary acts and attitudes that human agents perform and express. It is quite clear that propositions with the same truth conditions are not the senses of the same sentences, just as they are not the contents of the same thoughts. To account for that fact, the logic of propositions according to predication advocates finer criteria of propositional identity than logical equivalence and requires of competent speakers less than perfect rationality. Unlike classical logic it analyzes the structure of constituents of propositions. The logic is *predicative* in the very general sense that it analyzes the type of propositions by mainly taking into consideration the acts of predication that we make in expressing and understanding them. Predicative logic distinguishes strictly equivalent propositions whose expression requires different acts of predication or whose truth conditions are understood in different ways. It also explicates a new relation of strong implication between propositions much finer than strict implication and important for the analysis of psychological and illocutionary commitments. The main purpose of this work is to present and enrich the logic of proposi-

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tions according to predication by analyzing elementary propositions that predicate all kinds of attributes (extensional or not) as well as modal propositions according to which it is necessary, possible or contingent that things are so and so. I will first explain how predicative logic analyzes the structure of constituents and truth conditions of propositions expressible in the modal predicate calculus without quantifiers. The ideal object language of my logic is a natural extension of that of the minimal logic of propositions.[1] Next I will define the structure of a model and I will formulate an axiomatic system. At the end I will enumerate important valid laws. The present work on propositional logic is part of my next book *Propositions, Truth and Thought* which formulates a more general logic of propositions according to predication analyzing also generalization, ramified time, historic modalities as well as action and attitudes.

I will only discuss here modalities such as necessity, contingency and possibility as they are conceived in the broad logical universal sense of S5 modal logic.[2] All the truths of logic and mathematics are necessarily true in this wide sense[3] as are a lot of other analytically true propositions e.g. that husbands are married as well as some synthetically true propositions e.g. that whales are mammals. As Leibnitz pointed out[4], in asserting modal propositions, we consider possible worlds different from the real world in which we are. In the philosophical tradition, the *real world* is just the way things are, while a *possible world* is a way things could be. On one hand, a proposition is necessarily (or possibly) true in the broad logical sense when that proposition is true at all moments in all possible worlds (or at some moment in some possible world). On the other hand, a proposition is contingently true (or false) in the same sense when that proposition is true (or false) at a moment in the real but not in all possible worlds. From this point of view, in thinking that some propositions are logically necessary, possible or contingent we simply proceed to a universal or existential quantification over the set of all *possible circumstances* which are conceived here simply as pairs containing a moment of time and a possible world.

In order to analyze attributes and modalities, I will raise fundamental questions such as these: What is the nature of intensional attributes? What is the structure of constituents of elementary and complex propo-

[1]See my paper "A New Formulation of the Logic of Propositions" in M. Marion & R. Cohen (eds), *Québec Studies in the Philosophy of Science*, Volume 1, [1995]
[2]See C. I. Lewis, *A Survey of Symbolic Logic* [1918].
[3]See A. Plantinga, *The Nature of Necessity* [1974] for a philosophical explanation of the notions of logical necessity and possibility.
[4]See L. Couturat (ed.), *Opuscules et fragments inédits de Leibnitz* [1903].

sitions? In particular, which attributes do we predicate in expressing modal propositions? Moreover, how do we understand truth conditions? How are propositions related by the various kinds of implication (strict, analytic and strong implication) that we can distinguish in logic? We are not omniscient. We do not know the way things are in the real world. So we consider not only how things are but also how they could be. We conceive of many ways actual things could be and we can refer to possible objects which are not actual. We can also try to refer to objects which do not exist. We distinguish between certain necessarily true (or false) propositions and others which are contingently true (or false). Thus we know that it is necessary that $7 + 2 = 9$ and we think that it is contingent that there are nine planets. However we are sometimes inconsistent. We can assert and believe necessarily false propositions in science as well as in ordinary life. We used to believe the paradoxical principle of comprehension in naïve set theory. Some of us still believe that whales are fishes. Are there necessarily true propositions that we *a priori* know and necessarily false propositions that we could not believe? We do not draw all logical inferences. We can believe in the truth of incompatible propositions but these beliefs clearly do not commit us to believing any proposition whatsoever. What kinds of valid theoretical inferences are we able to make by virtue of linguistic competence? We, human agents are *minimally rational*[5] and *paraconsistent* in the use of language and the conduct of thought. Could we explicate rigorously minimal rationality in logic?

1. Principles of the logic of propositions according to predication

As is well known, so called *strictly equivalent* propositions (propositions which are true in the same possible circumstances) are not substitutable *salva felicitate* within the scope of illocutionary forces and psychological modes. We can assert (and believe) that Brasilia is a city without *eo ipso* asserting (and believing) that Brasilia is a city and not an erythrocyte. However, these two assertions (and beliefs) have strictly equivalent propositional contents; they are true under the same conditions. From a philosophical point of view, then, propositional identity requires more than truth in the same possible circumstances. We need a criterion of propositional identity stronger than strict equivalence in

[5]The term of *minimal rationality* comes from C. Cherniak *Minimal Rationality* [1986]

logic. It is a mistake to identify as Carnap advocated[6] each proposition with the set of possible circumstances in which it is true. On the basis of speech act theory, I advocate a finer analysis in terms of predication of the logical type of propositions. As I have pointed out repeatedly, even the simplest elementary propositions whose attribute is extensional and their truth functions have a more complex logical structure than truth conditions. Here are the basic principles of my theory of sense and denotation.[7]

1.1 A finite structure of constituents

Propositions are complex senses provided with a finite structure of constituents. As Frege, Russell, Strawson and many others pointed out, understanding a proposition consists mainly of understanding which *attributes* (properties or relations) objects of reference must possess in order that this proposition be true in a possible circumstance. In expressing and understanding propositions we *predicate* attributes *of* objects in a certain order. Propositional contents are then composed from a finite positive number of *atomic propositions* corresponding to *acts of predication.* Thus the proposition that Paul is wounded and smaller than Mary has two atomic propositions: one predicates of Paul the property of being wounded, the other predicates successively of Paul and Mary the relation of being smaller than.[8]

1.2 No singular propositions

Propositional constituents are senses and not pure denotations. As Frege[9] pointed out, we always *refer to* objects by subsuming them under senses. We cannot have directly in mind *individuals* which are objects of reference of the simplest type.[10] (Persons and material objects of the world which exist in space time are individuals.) We have in mind

[6]Classical logic follows R. Carnap *Meaning and Necessity* [1956]. See R. Barcan Marcus, *Modalities* [1993] and R. Montague, *Formal Philosophy* [1974]

[7]See "Universal Grammar and Speech Act Theory" in D. Vanderveken & S. Kubo (eds.) *Essays in Speech Act Theory* [2001] for propositional universals and *Formal Ontology, Propositional Identity and Truth According To Predication With an Application of the Theory of Types to the Logic of Modal and Temporal Proposition* in *Cahiers d'Épistémologie* [2003] for a more general presentation and axiomatization of my theory.

[8]Predication as it is conceived here is purely propositional and independent on force. To predicate a property of an object is not to judge that it has that property. It is just to *apply* the property to that object in the sense of functional application. We make the same predication when we assert and deny that an object has a property.

[9]See "On Sense and Reference" in P. Geach & M. Black (eds) *Translations from the Philosophical Writings of Gottlob Frege* [1970]

[10]See P.F. Strawson *Individuals* 1959.

concepts of such individuals and we *indirectly* refer to them through these concepts. So expressions used to refer to individuals have a sense called an *individual concept* in addition to sometimes a denotation in each context. When we speak literally we express the proposition that is the sense of the sentence used in the context of utterance. In that case we refer to the objects which fall under the concepts expressed by the referential expressions that we use. It can happen that there are no such objects. This does not prevent us from expressing a proposition. By recognizing the indispensable role of concepts in reference, logic can account for the meaning and referential use of proper names and definite descriptions without a denotation. They contribute to determine propositions which have (according to Russell) or lack (according to Frege and Strawson) a truth value in the context of utterance.

Frege's argument in favor of indirect reference remains conclusive if one accepts that every proposition is the possible content of a thought. From a cognitive point of view, it is clear that the proposition that the morning star is the morning star is very different from the proposition that the morning star is the evening star. We *a priori* know by virtue of linguistic competence the truth of the first proposition while we *a posteriori* learned the truth of the second at a certain period of history. A similar difference of cognitive value exists between the two propositions that Hesperus is Hesperus and that Hesperus is Phosphorus expressed by using the proper names "Hesperus" and "Phosphorus" of the morning star and the evening star respectively.[11] Frege's idea that propositional constituents are the senses and not the denotations of the expressions that we use to refer clearly explains the difference in cognitive value between the two propositions. It also preserves the minimal rationality of speakers. We can make mistakes and believe, as did the Babylonians, that the morning star is not the evening star or that Hesperus is not Phosphorus. But we could not assert or believe the contradictory proposition that the morning star is not the morning star or that Hesperus is not Hesperus. Otherwise we would be totally irrational. So logic has to reject the theory of *direct reference*[12] according to which certain referential expressions, logical proper names (according to the first Wittgenstein and Russell) ordinary proper names (according to Kaplan and Kripke), do not have any sense. There are *no singular propositions* having pure individual objects as constituents in the formal ontology that I advocate contrary to Russell, Quine, Davidson, Kaplan, Kripke

[11] The example was given by David Kaplan in a lecture at McGill University.

[12] The notion of direct reference comes from David Kaplan "On the Logic of Demonstratives", *Journal of Philosophical Logic* [1970].

and others who defend direct reference and externalism. Any *object of reference* is subsumed *under a concept*. Often proper names are introduced into language by an initial declaration.[13] A certain speaker gives the name to an object with which he is acquainted or that he discovers. And the name is adopted by the linguistic community which keeps using it to refer to the same object. Later speakers who do not know much of that object can always refer to it under the concept of being the object called by that name (this is their concept).[14] All propositional constituents are therefore senses: they are concepts or attributes.

1.3 Reference and predication

In my view, as in the logical tradition of Frege, Church, Carnap and Strawson, the two kinds of propositional constituents serve different roles in the determination of truth conditions: *attributes* serve to *predicate* while *concepts* serve to *refer* to objects. Attributes of individuals of degree n are *senses of* n-ary *predicates* while individual concepts are *senses of individual terms* in the formal semantics of the logic of propositions according to predication. So the domain of any possible interpretation of language contains a non empty set *Individuals* of individual objects as well as two non empty sets *Concepts* of individual concepts and *Attributes* of attributes of individuals.

1.4 A relation of correspondence between senses and denotations

There is a fundamental logical *relation of correspondence* between senses and denotations[15] underlying the relation of correspondence between words and things in philosophy of language. To propositional constituents *correspond actual denotations* of certain types in possible circumstances. Thus to each individual concept corresponds in each circumstance the single individual object which falls under that concept in that circumstance whenever there is such an object. Otherwise that concept is deprived of denotation in that circumstance. To each property of individuals corresponds in each circumstance the set of objects under concepts which possess that property in that circumstance. Individual

[13]See S. Kripke *Naming and Necessity* [1980]

[14]The fact that different speakers using a proper name to refer to an object can have very different private mental representations and sensorial impressions of that object as well as very different beliefs about it does not prevent them to have in mind a common concept of that object, for example, the object named by that name in current discourse.

[15]See A. Church "A Formulation of the Logic of Sense and Denotation" in P. Henle & *al* (eds) *Structure, Method and Meaning* [1951]

things change in the possible courses of history of the world. Their properties vary at different moments. So different denotations can correspond to the same concept or attribute in different circumstances. Few senses have a rigid denotation. However individual objects have certain unique *essential properties* (Plantinga 1974) in all circumstances where they exist. For example, each human being has his own genetic code. Speakers who refer to individuals do not know all essential properties.

As is well known, one must take into account the order in which we predicate a relation of several objects of reference. Many relations are not symmetric. Some are even asymmetric. This is why the denotation of a relation of degree n is a *sequence* of n objects under concepts. The order in the sequence shows the order of predication. The first, second, . . . , and last element of the sequence are the first, second, . . . , and last object of which the relation is successively predicated.

1.5 Intensional attributes

As is well known, many attributes that we predicate of objects of reference are *intensional*; they are satisfied by sequences of objects under certain concepts and unsatisfied by the same sequences of objects under other concepts. One can admire Napoleon under one concept (the winner of the battle of Austerlitz) without admiring him under another concept (the first Emperor of France). For that reason, the actual denotation of a first order attribute of individuals in a circumstance is a set of sequences of individuals under concepts rather than a set of sequences of individual objects. So logic can account for the predication of so-called *intensional attributes* and explain failures of the law of extensionality. *Extensional properties* like the property of being alive have a special feature; they cannot be possessed by an individual object under a concept in a circumstance without being also possessed by the same object under all other concepts of that object in that circumstance. So the truth value of an atomic proposition predicating an extensional property of an object under concept only depends on the denotation of that concept.

1.6 Ignorance of actual denotations

Our knowledge of the world is partial. We do not know by virtue of linguistic competence actual denotations of most propositional constituents in possible circumstances that we consider. We often *refer* to an object under a concept without knowing and being able to identify that object. The police officer who is pursuing the murderer of a certain person called Smith can just refer to whoever in the world is that murderer. Any speaker who refers to an object under a concept *presupposes* that a single object falls under that concept in the context of utterance. The concept gives *identity criteria* for the object of reference (e.g. to be Smith's murderer). But few identity criteria enable us to *identify* the object of reference. Moreover some of our beliefs are false. We can wrongly believe that an object falls under a concept. The presumed murderer is sometimes innocent. In that case the *object of reference* is not the *denotation* of the concept that we have in mind. It can also happen that no object satisfies the identity criteria. [16] (Suppose that Smith's death was accidental.) Or even that several objects satisfy them. (Smith was killed by several men.)

1.7 Many possible denotation assignments to senses

We can ignore who has killed a certain person. But we can at least think of different men who could have committed the crime. Whoever conceives propositional constituents can in principle assign to them possible denotations of appropriate type in circumstances. Our possible denotation assignments to senses are functions that associate with individual concepts one or no individual object at all and with attributes of degree n a set of n-ary sequences of individuals under concepts in possible circumstances. From a cognitive point of view, we often believe that only certain entities could be the denotations of attributes and concepts in circumstances that we consider. Certain possible valuations of propositional constituents are then incompatible with our beliefs. Suppose that the chief of police believes at the beginning of his investigation that Smith's murderer is either Paul or Julius. Then only possible denotation assignments according to which one of these two individuals falls under the concept of being Smith's murderer in relevant circumstances are then compatible with the beliefs of that chief during that period of his investigation.

Among all possible valuations of propositional constituents **there is of course a special one, the *real valuation* (in symbol *val**), that associates with each concept and attribute its actual denotation in any possible circumstance**. Actual circumstances represent a complete state of the actual world at a moment. Possible circum-

[16]Notice that the property of existence is a second order property. The speaker who says that the Golden Mountain does not exist does not refer to that mountain. He could not presuppose the existence of that mountain since he is denying that it exists. So the property of existence does not apply to individual objects under concepts but rather to individual concepts themselves. That property is satisfied by an individual concept in a circumstance when that concept applies to one individual in that circumstance.

stances whether actual or not belong to the logical space of *reality*. We ignore how things are in actual and other possible circumstances of the reality. So we cannot determine which possible valuation is the *real one*. Consider the atomic proposition that attributes to Smith's murderer the property of being wounded. Its concept and property could have many different denotations. According to a first possible denotation assignment a suspect Paul would be Smith's murderer and that suspect would also be wounded in the present circumstance. According to a second, a thief Julius would be the murderer but he would not be wounded now. According to a third no one would have killed Smith. Clearly we need more than linguistic knowledge in order to determine the actual denotation of the concept of being Smith's murderer and the property of being wounded in the actual world. An empiric investigation is required to get that knowledge. However we all know *a priori* according to which possible denotation assignments to its constituents the atomic proposition is true. We know that it is true in actual circumstances according to the first possible denotation assignment considered above and false according to the two others. We also know by virtue of competence that in order to be *true* an atomic proposition must be true according to possible valuations of its constituents which correspond to reality. So we know that the atomic proposition above is true in the present circumstance if and only if a single person really killed Smith and is also wounded now. It does not matter whether or not we know who that person is.

1.8 Meaning postulates

We respect *meaning postulates* in assigning possible denotations to senses and truth conditions to propositions. We assign to propositional constituents denotations of appropriate type. As I said in the last section, possible valuations of propositional constituents associate with each individual concept c_e and possible circumstance c a single individual object or no individual at all. Thus $val(c_e, c) \in Individuals$ or val is undefined for the concept c_e in the circumstance c. In that case, I will for the sake of simplicity like Carnap [1956] identify $val(c_e, c)$ with an arbitrary entity, the empty individual $u_\varnothing$ (rather than the empty set $\varnothing$). The *empty individual* is conceived here as the individual that does not exist at any moment in any possible world.

Possible valuations associate with each attribute R_n of degree n of individuals and possible circumstance a set of n-ary sequences of individuals under concepts. So $val(R_n, c) \in \mathscr{P}(Concepts^n)$. We moreover respect the logical nature of concepts and attributes and internal relations that exist between them because of their logical form. For we apprehend that logical form in conceiving them. Individuals subsumed under two concepts are identical when these two concepts have the same denotation. So the denotation in each possible circumstance of the binary relation of identity between individuals $\| = \|$ is the same according to all possible valuations of senses: it is the set of all pairs of individual concepts applying to the same individual in that circumstance. $<c_e^1, c_e^2> \in val(\| = \|, c)$ when $val(c_e^1, c) = val(c_e^2, c)$. We know *a priori* by virtue of linguistic competence that objects which fall under certain concepts (e.g. the concept of being Smith's murderer) have therefore certain properties (e.g. to be a murderer). And that they could not possess certain properties (to be admirable) without having others (to be admired in a possible circumstance). So possible valuations of logical constants respect traditional meaning postulates.

1.9 Truth according to a possible denotation assignment to constituents

By definition, an atomic proposition of the form $(R_n(c_e^1, \ldots, c_e^n))$ predicating the attribute R_n of n individuals under concepts $c_e^1, \ldots, c_e^n$ in that order is true in a circumstance according to a possible valuation when the sequence of these objects under concepts $c_e^1, \ldots, c_e^n$ belongs to the denotation that that valuation assigns to its attribute in that circumstance. So every possible valuation *val* of propositional constituents associates certain *possible truth conditions* with all atomic propositions containing such constituents. Any atomic proposition of the form $(R_n(c_e^1, \ldots, c_e^n))$ is true in a circumstance c according to a possible valuation *val* of its constituents when $<c_e^1, \ldots, c_e^n> \in val(R_n, c)$. Otherwise it is false in that circumstance according to that valuation.

1.10 Possible truth conditions

Because we ignore actual denotations of most propositional constituents, we also ignore in which possible circumstances most atomic propositions are true. We just know that they could be true in different sets of possible circumstances given the various denotations that their senses could have in the reality. For that reason, in my approach, propositions have *possible truth conditions* in addition to actual Carnapian *truth conditions*. For any atomic proposition one can distinguish as many possible truth conditions as there are distinct sets of possible circumstances where that atomic proposition is true according to a possible denotation assignment to its propositional constituents. Suppose that an atomic proposition is true in a set of possible circumstances according to a certain possible valuation of its constituents. Then clearly it would be true in all and only these circumstances if that valuation of these constituents *were real*, that is to say if it were associating with them their actual denotations. So the corresponding set of these possible circumstances corresponds well to a certain possible truth condition of that atomic proposition. As one can expect, every possible complete valuation of propositional constituents *determines* a unique possible complete valuation of atomic propositions. It assigns to them in accordance with meaning postulates *possible truth conditions* that they could all have together.

In my approach, there are a lot of *subjective* in addition to *objective possibilities* in the reality. When a possible denotation assignment *val* is compatible with the beliefs of an agent in a circumstance, any atomic proposition which is true in that circumstance according to that assignment, is then a proposition that could be true according to him or her in that circumstance. So, for example, according to the chief of police above at the beginning of his investigation Paul could be Smith's murderer.

1.11 Actual truth conditions

Among all possible truth conditions of an atomic proposition there are of course its *actual characteristic* Carnapian *truth conditions* that correspond to the set of possible circumstances where it is true.[17] So among all possible valuations of atomic propositions there is also a special one, let us call it the *real valuation*, that associates with each atomic proposition its actual truth conditions. As one can expect, that *real valuation* of *atomic propositions* is determined by the *real valuation val** *of propositional constituents* that we have distinguished above: the one which assigns to each concept and attribute its actual denotation in each possible circumstance. An atomic proposition *is true in a circumstance* when it is true in that circumstance according to all possible valuations of senses that associate with its propositional constituents their actual denotation in the reality. For in that case the sequence of its objects under concepts in the order of predication belongs to the actual denotation of its attribute in each possible circumstance.

1.12 The type of atomic propositions

We can ignore in which circumstances an atomic proposition is true. But we could not *apprehend* one without having in mind its propo-

[17]Carnap did not consider possible truth conditions other than actual truth conditions.

sitional constituents: its single main attribute of degree n and the n individual concepts under which are subsumed the objects of reference. And without knowing under which conditions that atomic proposition is true. From a logical point of view, each atomic proposition of the form $(R_n(c_e^1, \ldots, c_e^n))$ is then a pair whose first element is the set of its n + 1 propositional constituents and whose second element is the set of all possible circumstances where it is true. In symbols, $(R_n(c_e^1, \ldots, c_e^n)) = \langle \{R_n, c_e^1, \ldots, c_e^n\}, \{c / \langle c_e^1, \ldots, c_e^n \rangle \in val^*(R_n, c)\}$ where *val** is the real valuation. Notice that the order of predication only matters when it affects truth conditions. The propositions that Hesperus is Phosphorus and that Phosphorus is Hesperus do not differ. For the relation of identity is symmetric. We all know that by virtue of competence.

1.13 A recursive definition of propositions

In my analysis, complete propositions have then *a structure of constituents*: they are composed from a finite positive number of atomic propositions. They also have *possible truth conditions*: they are true in certain sets of possible circumstances according to possible valuations of their constituents. Until now I have mainly analyzed atomic propositions which are the basic units of the structure of constituents of propositions. One can define recursively the set of complete propositions that are expressible in the present modal logic. *Elementary propositions* are the simplest propositions: they are composed from a single atomic proposition and have all its possible truth conditions. Other more complex propositions are obtained by a finite number of applications of truth functional and modal operations to simpler propositions. Complex propositions can be composed from several atomic propositions and, when they are composed from a single atomic proposition, they do not have the same possible truth conditions.

What is the structure of constituents of truth functions and modal propositions? Which attributes do we predicate in expressing them? And how do we determine their possible truth conditions from the possible truth conditions of their constituent atomic propositions?

1.14 Structure of constituents of truth functions

As Wittgenstein pointed out in the *Tractatus,* truth connectives do not serve to make new acts of reference or predication. Truth functions do not change the structure of constituents. Their meaning just contributes to determining truth conditions. Truth functions of various propositions are composed from all and only the atomic propositions of their arguments. Thus the *negation* ¬P of a proposition P is composed from the atomic propositions of P. The *conjunction* $(P \wedge Q)$ and the *disjunction* $(P \vee Q)$ of two propositions P and Q are composed from the atomic propositions of both.

1.15 Structure of constituents of modal propositions

Unlike truth connectives, modal connectives serve to make new predications of so called *modal attributes.* Their meaning contributes to changing both the structure of constituents and the truth conditions of propositions. In thinking the modal proposition that it is impossible that God makes mistakes we do more than predicate of God the property of not making mistakes. We also predicate of Him the modal property of infallibility namely that He does not make a mistake in any possible circumstance. Infallibility is the necessitation of the property of not making mistakes. Modal proposition are then composed from new atomic propositions predicating modal attributes of some of their objects under concept.

Contrary to what Jorge Rodriguez[18] thinks, there is no need to enter into the infinite set of ramified types of propositions in order to analyze in terms of predication the attributes of modal propositions. The new attributes of modal propositions according to which it is necessary that P (in symbols $\Box P$) or that it is possible that P (in symbols $\Diamond P$) remain of the first order. In expressing these modal propositions we do not predicate of their argument, proposition P, the second order modal property of being true in all (or in some) possible circumstances. Rather we predicate corresponding modal attributes of objects under concepts of that argument. In the logic of attributes,[19] modal attributes of individuals are obtained from simpler attributes by quantifying universally or existentially over possible circumstances. The two basic kinds of broad modal operations on attributes associate with any given attribute the *necessitation* and the *possibilization* of that attribute. By definition, an object under concept possesses the necessitation of a property when it possesses that property in all possible circumstances. And it possesses the possibilization of a property when it possesses that property in at least one possible circumstance. (And similarly for

[18] J. Rodriguez Marqueze, "On the Logical Form of Propositions: Some Problems for Vanderveken's New Theory of Propositions" in *Philosophical Issues* [1993].
[19] See G. Bealer *Quality and Concept* [1982].

relations.) Suffixes like "ible" and "able" serve to compose modal predicates in English. Thus the property of being perturbable is the possibilization of the property of being perturbed. Someone is perturbable when he is perturbed in at least one possible circumstance. I will also use the logical constants $\Box$ and $\Diamond$ to express modal attributes. In my symbolism $\Box\mathrm{R}_n$ and $\Diamond\mathrm{R}_n$ are respectively the *necessitation* and the *possibilization* of the attribute R_n. By definition, all possible valuations of propositional constituents respect the following meaning postulates: $<c_e^1, \ldots, c_e^n> \in val(\Box R_n, c)$ when, for every c', $<c_e^1, \ldots, c_e^n> \in val(R_n, c')$. And similarly, $<c_e^1, \ldots, c_e^n> \in val(\Diamond R_n, c)$ when, for at least one c', $<c_e^1, \ldots, c_e^n> \in val(R_n, c')$.

So the formal ontology that I advocate here remains simple. There are only individuals under concepts, attributes of such individuals and first order atomic propositions containing such propositional constituents. There is no ramification of the logical type of propositions. All the modal attributes of the form $\Box R_n$ and $\Diamond R_n$ are of the first order: they are satisfied by (sequences of) individuals under concepts and not by propositions. On the basis of such considerations one can define simply the structure of constituents of modal propositions. A modal proposition of the form $\Box$P or $\Diamond$P contains in addition to any atomic proposition of its argument P predicating an attribute R_n of n individuals under concepts two new atomic propositions predicating in the same order the necessitation $\Box R_n$ and the possibilization $\Diamond R_n$ of that attribute[20] of the same individuals.[21]

1.16 Understanding of truth conditions

How do we understand the truth conditions of propositions? As Wittgenstein pointed out[22], in understanding the conditions under which a proposition is true, we always distinguish between different possible ways in which its objects might be, those which are

[20] There are four modal attributes corresponding to the modal operations of S5 modal logic namely $\Box R_n$, $\Box\neg R_n$, $\Diamond R_n$ and $\Diamond\neg R_n$ where possibility $\Diamond$ is defined as $\neg\Box\neg$. However, the operations of necessitation $\Box$ and possibilization $\Diamond$ are sufficient for my purposes here. For all modal propositions MP where M = $\Box$, $\Box\neg$, $\Diamond$ or $\Diamond\neg$ have the same structure of constituents, no matter how many modal attributes are taken into consideration.

[21] As one can expect, there are four different basic modal functions of a proposition P, namely: $\Box$P, $\Box\neg$P, $\Diamond$P and $\Diamond\neg$P corresponding to the four basic types of modal attributes $\Box R_n$, $\Box\neg R_n$, $\Diamond R_n$ and $\Diamond\neg R_n$ which can be formed from any attribute R_n in the logic of attributes.

[22] See aphorisms 4.3 and 4.4 of the *Tractatus logico-philosophicus*.

compatible with its truth from those which are not. In my approach, we distinguish in understanding a proposition P between two kinds of possible ways in which its propositional constituents might correspond to reality, those according to which P is true from those according to which it is false. In making such a distinction we consider all the atomic propositions of P and draw a large *truth table* more complex than that of Wittgenstein. In the *Tractatus* all propositional constituents are individual objects which are pure denotations. In my logic, they are senses: concepts and attributes to which correspond objects and concepts of objects respectively. Moreover, not all propositions are truth functions. There are modal propositions. So we have to distinguish in drawing a truth table for a proposition P two disjoint sets of possible valuations of its constituents *with respect to* one or more possible *circumstances*: those that assign to atomic propositions possible truth conditions that are compatible with the truth of P in these circumstances from those which do not.

Let me explain this by induction. By definition, an *elementary proposition* is true in a circumstance according to a possible valuation of its constituents when that valuation associates with its attribute in that circumstance a denotation that contains the sequence of its objects under concepts in the order of predication. This is the way objects have to be in order that its single atomic proposition be true according to a valuation in a circumstance. So the possible truth conditions of an elementary proposition are the possible truth conditions of its unique atomic propositions. As one can expect, the negation ¬P is true in a circumstance according to a possible valuation of its constituents when the proposition P is false according to that valuation in that circumstance. In other words, the truth of proposition ¬P in a circumstance is only compatible with possible truth conditions of its atomic propositions that are incompatible with the truth of P in that very circumstance. Furthermore, a conjunction (P ∧ Q) is true in a circumstance *c* according to a possible valuation when both conjuncts P and Q are true in *c* according to that valuation. So the truth of a conjunction in a circumstance is only compatible with possible truth conditions of its atomic propositions that are compatible with the truth of both conjuncts P and Q in that circumstance. Truth functions obey the law of extensionality. Their truth value in a circumstance according to a valuation only depends on the truth value of their arguments in that circumstance according to that valuation. On the contrary, modal operations are intensional. A modal proposition of the form $\Box$P (or $\Diamond$P) is true in a possible circumstance according to a possible valuation of its constituents when its argument P is true in every (or in at least one) possible circumstance c' according to that valuation. So the truth of modal propositions $\Box$P (or $\Diamond$P) in a circumstance is only compatible with possible truth conditions of its atomic propositions that are compatible with the truth of its argument P in every (or at least one) possible circumstance.

1.17 Tautologies and contradictions

There are two borderline cases of truth conditions. In the first case, the truth of a proposition is compatible with all the possible ways in which objects might be. It is a *tautology.* In the second case, its truth is not compatible with any possible way in which objects might be. It is a *contradiction.* In my approach, tautologies are true according to all possible valuations of their constituents while contradictions are true according to none. So the truth of a *tautology* in any possible circumstance is compatible with all the possible truth conditions of its atomic propositions, and the truth of a *contradiction* with none. For that reason, tautologies (and contradictions) are a very special case of necessarily true (and necessarily false) propositions. When we express a tautology and a contradiction we *a priori* know in apprehending their logical form that the first is necessarily true and the second is necessarily false. Tautologies are then unconditionally, *a priori* and analytically true, *contradictions* unconditionally, *a priori* and analytically false.

1.18 The new criterion of propositional identity

Identical propositions have the same structure of constituents and they are true in the same possible circumstances according to the same possible denotation assignments to their propositional constituents. My criterion of propositional identity is much finer than that of modal, temporal, intensional and relevance logics. My logic distinguishes strictly equivalent propositions composed of different atomic propositions. We clearly do not make the same predications in expressing them. So we do not have them in mind in the same possible contexts of utterance. There are a lot of different necessarily true and necessarily false propositions and not only two as classical logic wrongly claims. Tautologies with different constituents are different propositions.

Predicative logic moreover distinguishes strictly equivalent propositions with the same structure of constituents which are not true in the same circumstances according to the same possible valuations of their constituents. When the truth of two propositions is not compatible with the same possible truth conditions of their atomic propositions, we indeed do not understand their truth conditions in the same way. Consider the elementary proposition that the biggest whale is a fish and the conjunction that the biggest whale is and is not a fish. Both are composed from the same atomic proposition predicating of the biggest whale the property of being a fish. And both are necessarily false. In all possible circumstances where they exist, whales are mammals. They all have in common that *essential property*. However the two propositions have a different cognitive value. We recently discovered that whales are mammals. Previously we had believed that the biggest whale was a fish. But we could never have believed that it is and that it is not a fish. Unlike Parry's logic of analytic implication my predicative logic distinguishes such strictly equivalent propositions with the same structure of constituents. Clearly the elementary proposition that a whale is a fish is necessarily false. However it is true according to many possible valuations of its constituents (all those according to which the denotation of the property of being a whale is a subset of the denotation of being a fish). On the contrary, the proposition that a whale is and is not a fish is a pure contradiction: it is not true according to any possible valuation of its constituents. This is why we cannot believe it.

When two propositions are true in the same possible circumstances according to the same possible denotation assignments to their propositional constituents, their truth in each circumstance is by hypothesis compatible with the same possible truth conditions assignments to their atomic propositions. Possible valuations of propositional constituents *determine* by definition all possible valuations of atomic propositions. Thus from a logical point of view one can identify each proposition P with a pair whose first element is the finite non empty set of its atomic propositions and whose second element is the function associating with any possible circumstance the set of possible valuations of its atomic propositions which are compatible with its truth in that very circumstance. Propositions belong to the set $\mathscr{P}\mathrm{U}_a\times$ (*Circumstances* $\Rightarrow \mathscr{P}(\mathrm{U}_a \Rightarrow \mathscr{P}$*Circumstances*). My theory of sense and propositions is compatible with the current dynamic analysis of meaning according to which the meaning of a sentence in a context of utterance is related to information change potential.[23]

1.19 Truth definition

In the philosophical tradition, from Aristotle to Tarski, *truth* is based on *correspondence* with reality. True propositions represent how objects are in the reality. Objects of reference have properties and stand in relations in possible circumstances. Atomic propositions have therefore a well determined truth value in each circumstance depending on the denotation of their attributes and concepts and the order of predication. However things could have many other properties and stand in many other relations in each circumstance. In addition to the ways in which things are, there are the possible ways in which they could be. Our knowledge is restricted. So we consider a lot of possible truth conditions of atomic propositions different from their actual truth conditions in thinking propositional contents. In our mind, the truth of propositions is compatible with many possible ways in which objects could be. However in order that a proposition be true in a given circumstance, things must be in that circumstance as that proposition represents them. Otherwise, there would be no correspondence. Along these lines, one can say that a proposition *is true in a possible circumstance* when it is true according to any *real* valuation of its propositional constituents assigning to them their actual denotation in each circumstance. In that case its truth in that circumstance is compatible with the actual truth conditions of all its atomic propositions. So a proposition P is true in a circumstance c when it is true according to the real valuation of propositional constituents, that is to say when *val**

[23]Each new sentence in a discourse has to be interpreted in the conversational background of the context in which it is uttered and its interpretation (the illocution that it expresses in that context) updates that background. For the principles of my semantic theory see my paper "Success, Satisfaction and Truth in the logic of Speech Acts and Formal Semantics" in S. Davis & B. Gillan *A Reader in Semantics* [2004]

$\in$ id$_2$P(c). Classical laws of truth theory follow from this concise definition.

1.20 Cognitive aspects in the theory of truth

Each agent *a* has in mind a finite number of propositional constituents in each circumstance c and what he then believes depends on the possible denotations that these constituents have or could have according to him in the reality. So to each agent *a* and circumstance c there corresponds a unique set *Val(a,c)* containing all the possible valuations of senses *compatible with what that agent believes in that circumstance*. Suppose that an agent *a* believes in a circumstance c that no individual could fall under a concept c_e. Then according to all valuations $Val \in Val(a,c)$ compatible with what he then believes, $Val(c_e,c) = u_\varnothing$ for any possible circumstance *c*. Any agent having in mind propositional constituents *believes* in the truth of certain propositions containing them. One can now define adequately the notion of *belief* in philosophical logic: *an agent a believes a proposition in a circumstance c* when firstly, that agent has then in mind all its propositional constituents and secondly, that proposition is true in that circumstance according to all possible valuations of constituents f $\in$ *Val(a,c)* that are compatible with his beliefs in that circumstance.[24] As one can expect, tautological propositions are true and contradictory propositions are false according to all agents who have them in mind. But impossible propositions which are not contradictory can be true and necessary propositions which are not tautological can be false according to agents at some moments. These are basic principles of my epistemic logic. So the logic of language imposes different limits on experience and thought. *Objective* and *subjective possibilities* differ. Necessarily false propositions represent impossible facts that could not exist in reality and that we could not experience. In my view there is no need to postulate impossible circumstances where such impossible facts would exist. Impossible facts are objectively impossible. In any possible circumstance where there are whales they

[24]Whenever an agent does not think or act at all (he is in a profound sleep or dead), all possible valuations of propositional constituents are then compatible with his beliefs. But he does not then believe anything by hypothesis. In order to have a conscious belief an agent must have in mind relevant concepts and attributes.

are mammals and not fishes. However there are many more subjective than objective possibilities. Certain objectively impossible facts e.g. that whales are fishes are subjectively possible. Their existence is compatible with certain possible denotation assignments to senses. So we can wrongly believe that exist.

1.21 The notion of strong implication

We, human beings are not perfectly rational. Not only do we make mistakes and have a lot of false beliefs. But we are often inconsistent. Moreover we do not draw all valid inferences. So we assert (and believe) propositions without asserting (and believing) all their logical consequences. Our illocutionary (and psychological) commitments are not as strong as they should be from the logical point of view. We do not even know all logical truths. However we are not completely irrational. On the contrary, we manifest a *minimal rationality* in thinking and speaking that logic can now explain. We know that certain propositions are necessarily false (for example, contradictions): we cannot believe them nor intend to bring about facts that we know to be impossible.[25] Moreover, we always draw certain valid theoretical inferences. When we know *a priori* by virtue of competence that a proposition cannot be true unless another is also true, we cannot believe (or assert) that proposition without believing (or asserting) the other. There is an important *relation of strict implication* between propositions due to C.I. Lewis that has been much used in epistemic logic: a proposition *strictly implies* another whenever that proposition cannot be true in a possible circumstances unless the other is true in that same circumstance. Hintikka[26] and others claim that belief and knowledge are closed under strict implication. However we ignore which propositions are related by strict implication, just as we ignore in which possible circumstances they are true. Moreover we could not know all cases of strict implication. For any proposition strictly implies infinitely many other propositions. We could not think of all of them in a context of utterance.

So we need a relation of propositional implication much finer than strict implication in order to explicate our illocutionary and psycho-

[25]See next chapter 15 “Attempt, Success and Action Generation” in this Volume.
[26]See J. Hintikka *Knowledge and Belief* [1962]

logical commitments. Predicative logic can define rigorously that finer propositional implication that I have called *strong implication.* By definition, *a proposition strongly implies another proposition when firstly, it contains all its atomic propositions and secondly, it tautologically implies that other proposition*: whenever it is true in a possible circumstance according to a possible valuation of its propositional constituents the other is also true in that circumstance according to the same valuation. Unlike strict implication, *strong implication is known.* Whenever a proposition P strongly implies another Q, we cannot express that proposition without knowing *a priori* that it strictly implies the other. For in expressing P, we have by hypothesis in mind all atomic propositions of Q. We make all the corresponding acts of reference and predication. Furthermore, in understanding the truth conditions of proposition P, we distinguish all possible valuations of its propositional constituents which are compatible with its truth in any circumstance. These are by hypothesis compatible with the truth of proposition Q in the same circumstance. Thus, in expressing P, we know that Q follows from P. Belief and knowledge are then closed under strong rather than strict implication in my epistemic logic. As I will show later, *strong implication obeys a series of important universal laws.* Unlike strict implication, strong implication is anti-symmetrical. Two propositions which strongly imply each other are identical. Unlike Parry's analytic implication, strong implication is always tautological. Natural deduction rules of elimination and introduction generate strong implication when and only when all atomic propositions of the conclusion belong to the premises. So a proposition P does not strongly imply a disjunction of the form P $\vee$ Q containing new constituents. Moreover strong implication is *paraconsistent.* A contradiction does not strongly imply all propositions. Finally, strong implication is both *finite* and *decidable.*

2. The ideal object-language

The object language $\mathscr{L}$ of my modal predicate calculus is an **extension of that of the minimal logic of propositions**.

2.1 Vocabulary of $\mathscr{L}$

(1) A series of *individual constants* called *individual terms*:
c, c′, c″, . . .
(2) for each positive natural number n, a series of *predicate constants of degree* n:
r_n, r'_n, r''_n, r'''_n, . . . including the binary identity predicate $=_2$
(3) the *syncategorematic expressions*:
=, >, ∧, ¬, □, ◊, [, (,] and).

2.2 Rules of formation of $\mathscr{L}$

Predicates

Every predicate of degree n of the lexicon is a predicate of degree n of $\mathscr{L}$. If R_n is a predicate of degree n, so are $\Box R_n$ and $\Diamond R_n$. Complex predicates of the forms $\Box R_n$ and $\Diamond R_n$ name respectively the modal attributes of degree n which are the *necessitation* and the *possibilization* of the attribute named by R_n.

The set L_a of predication formulas

If R_n is a predicate of degree n and t_1, . . . and t_n are n individual terms, then $(R_n t_1 \ldots t_n)$ is a predication formula which expresses the atomic proposition predicating the attribute expressed by R_n of the n individual concepts expressed by t_1, . . . and t_n in that order.

The set L_p of propositional formulas

If $(R_n t_1 \ldots t_n)$ is a predication formula then $[(R_n t_1 \ldots t_n)]$ is a propositional formula. If A_p and B_p are propositional terms, then $\neg A_p$, $\Box A_p$, $(A_p \wedge B_p)$, $(A_p > B_p)$ and $(A_p = B_p)$ are new complex propositional formulas. $[(R_n t_1 \ldots t_n)]$ expresses the *elementary proposition* whose unique atomic proposition is that expressed by predication formula $(R_n t_1 \ldots t_n)$. $\neg A_p$ expresses the *negation* of the proposition expressed by A_p. $\Box A_p$ expresses the *modal proposition* that it is logically necessary that A_p. $(A_p \wedge B_p)$ expresses the *conjunction* of the two propositions expressed by A_p and B_p. $(A_p > B_p)$ expresses the proposition according to which all atomic propositions of B_p are atomic propositions of A_p. Finally, $(A_p = B_p)$ means that propositions A_p and B_p are identical.

2.3 Rules of abbreviation

Parentheses are eliminated according to the usual rules.

Identity: $t_1 = t_2 =_{df} (=_2 t_1 t_2)$
Disjunction: $(A_p \vee B_p) =_{df} \neg(\neg A_p \wedge \neg B_p)$
Material implication: $(A_p \Rightarrow B_p) =_{df} \neg A_p \vee B_p$
Material equivalence: $(A_p \Leftrightarrow B_p) =_{df} (A_p \Rightarrow B_p) \wedge (B_p \Rightarrow A_p)$
Logical possibility: $\Diamond A_p =_{df} \neg\Box\neg A_p$
Strict implication: $A_p \multimap B_p =_{df} \Box(A_p \Rightarrow B_p)$
Tautologyhood: *Tautological*$(A_p) =_{df} A_p = (A_p \Rightarrow A_p)$
Analytic implication: $A_p \rightarrow B_p =_{df} (A_p > B_p) \wedge (A_p \multimap B_p)$
Analytic equivalence: $A_p \leftrightarrow B_p =_{df} (A_p \rightarrow B_p) \wedge (B_p \rightarrow A_p)$
Strong implication:
$A_p \mapsto B_p =_{df} (A_p > B_p) \wedge$ *Tautological* $(A_p \Rightarrow B_p)$
Same structure of constitutents:
$A_p \equiv B_p =_{df} (A_p > B_p) \wedge (B_p > A_p))$
Identical individual concepts: $\hat{}t_1 = \hat{}t_2 =_{df} [(r_1\ t_1)] > [(r_1 t_2)]$
Identical attributes: $\hat{}R_n = \hat{}R'_n =_{df} [(R_n\ t_1 \ldots t_n)] > [(R'_n\ t_1 \ldots t_n)]$
for the first n individual constants

3. The formal semantics

A *standard model* $\mathscr{M}$ for $\mathscr{L}$ is a sextuple < *Circumstances*, *Individuals*, *Concepts*, *Attributes*, *Val*, *, $\|\|$ >, where *Circumstances*, *Individuals*, *Concepts* and *Attributes* are four disjoint non empty sets, *Val* is a set of functions and * and $\|\|$ are functions which satisfy the following clauses:
(1) *Circumstances* is the set of *possible circumstances.*
(2) *Individuals* is the set of *individual objects.* For each possible circumstance *c*, $Individuals_c$ is the set of individual objects existing in that circumstance. Let $u_\varnothing$ be the *empty individual* of model $\mathscr{M}$. By definition,

$$Individuals = \bigcup_{c \in Circumstances} Individuals_c \cup \{u_\varnothing\}$$

(3) *Concepts* is the set of *individual concepts* and
(4) *Attributes* is the set of *attributes* of individuals considered in the model $\mathscr{M}$. For each positive natural number n, *Attributes* (n) is a non empty subset of *Attributes* containing all *attributes* of degree n considered in the model $\mathscr{M}$.
(5) $\|\|$ is an interpreting function which associates with each well formed expression *A* of $\mathscr{L}$ its semantic value $\|A\|$ in the model $\mathscr{M}$.

(i) For any individual constant t, $\|t\|$ is a certain individual concept $c_e \in$ *Concepts.*

(ii) For any predicate R_n of degree n, $\|R_n\|$ is a certain attribute of degree n $\in$ *Attributes* (n).

(6) *Val* is the set of all possible *assignments of denotation to propositional constituents* in the model $\mathscr{M}$. It contains a special *real valuation* val $\mathscr{M}$ which assigns to concepts and attributes their *actual denotation* in each possible circumstance according to the model $\mathscr{M}$. The set *Val* is the smallest subset of (*Concepts* $\cup$ *Attributes*) $\times$ *Circumstances* $\rightarrow$ (*Individuals* $\cup \bigcup_{1\leq n} \mathscr{P}(Concepts^n)$) which satisfies the following *meaning postulates*:

- For any valuation $val \in Val$ and possible circumstance c, *val* ($\|t\|$, c) $\in$ *Individuals* for any individual term t and *val* ($\|R_n\|$, c) $\in \mathscr{P}(Concepts^n)$ for any predicate R_n of degree n.
- $<\|t_1\|,\|t_2\|> \in val(\|=\|, c)$ iff $val(\|t_1\|, c) = val(\|t_2\|, c)$.
- $<\|t_1\|, \ldots,\|t_n\|> \in val(\|\Box R_n\|, c)$ iff, for every $c' \in$ *Circumstances*, $<\|t_1\|, \ldots,\|t_n\|> \in val(\|R_n\|, c')$.
- And similarly $<\|t_1\|, \ldots, \|t_n\|> \in val(\|\Diamond R_n\|, c)$ iff $<\|t_1\|, \ldots,\|t_n\|> \in val(\|R_n\|, c')$ for at least one possible circumstance c$'$.

(7) For any predication formula ($R_n t_1, \ldots, t_n$), $\|(R_n t_1, \ldots, t_n)\|$ is the *atomic proposition* predicating the attribute $\|R_n\|$ of the n objects under concepts $\|t_1\|, \ldots,\|t_n\|$ in that order. Formally, $\|(R_n t_1, \ldots, t_n)\|$ is the pair $<\{\|R_n\|,\|t_1\|, \ldots,\|t_n\|\}, \{c \in$ *Circumstances* / $<\|t_1\|, \ldots,\|t_n\|> \in$ val $\mathscr{M}(\|R_n\|, c)\}>$.

Let $U_a =_{def} \{ \|A_a\| / A_a \in L_a \}$ be the *set of all atomic propositions* considered in the model $\mathscr{M}$.

$\mathscr{P}[U_a]$ is an upper modal semi lattice containing finite sets of atomic propositions which is closed under union $\cup$ and a unary operation * satisfying the following clause: for any $\{\|(R_n t_1, \ldots, t_n)\|\} \in U_a$, $*\{\|(R_n t_1, \ldots, t_n)\|\} = \{\|(R_n t_1, \ldots, t_n)\|\}, \|(\Box R_n t_1, \ldots, t_n)\|, \|(\Diamond R_n t_1, \ldots, t_n)\|\}$ and, for any Γ_1 and $\Gamma_2 \in \mathscr{P}U_a$, $*(\Gamma_1 \cup \Gamma_2) = *\Gamma_1 \cup *\Gamma_2$ and $**\Gamma_1 = *\Gamma_1$. The elements of $\mathscr{P}[U_a]$ represent *structures of constituents* of propositions in the model $\mathscr{M}$.

(8) For any propositional formula A_p, $\|A_p\|$ is the *proposition* expressed by that formula according to the model $\mathscr{M}$. It belongs to the set $(\mathscr{P}U_a) \times$ (*Circumstances* $\Rightarrow \mathscr{P}$ *Val*). As one can expect, the first term, $id_1\|A_p\|$, of proposition $\|A_p\|$ represents the *set of its atomic propositions*. And its second term, $id_2\|A_p\|$, the way in which we understand its *truth conditions*, that is the function which associates with each possible circumstance c the set $id_2P(c)$ of all

possible valuations of propositional constituents according to which that proposition is true in that circumstance *c*.
The *proposition* $\|A_p\|$ expressed by A_p in the model $\mathscr{M}$ is defined by induction on the length of A_p:
Basis: $id_1\|[(R_n t_1, \ldots, t_n)]\| = \{\|(R_n t_1, \ldots, t_n)\|\}$ and $id_2(\|[(R_n c_1, \ldots, c_n)]\|, c) = \{val \in Val/\text{<}\|t_1\|, \ldots, \|t_n\|\text{>} \in val(\|R_n\|, c)\}$.
Induction steps:
(i) $id_1\|\neg B_p\| = id_1\|B_p\|)$ and $id_2(\|\neg B_p\|, \text{c}) = Val - id_2(\|B_p\|, \text{c})$.
(ii) $id_1\|\Box B_p\| = {}^{*}\, id_1\|B_p\|)$ and

$$id_2(\|\Box B_p\|, \text{c}) = \bigcap_{c' \in Circumstances} id_2(\|B_p\|, \text{c}')$$

(iii) $id_1(\|B_p \wedge C_p\|) = id_1(\|B_p\|) \cup id_1(\|C_p\|)$; $id_2(\|B_p \wedge C_p\|, \text{c}) = id_2(\|B_p\|, \text{c}) \cap id_2(\|C_p\|, \text{c})$.
(iv) $id_1(\|B_p \text{>} C_p\|) = id_1(\|B_p\|) \cup id_1(\|C_p\|)$ and $id_2(\|B_p \text{>} C_p\|, c) = Val$ when $id_1\|B_p\| \subseteq id_1\|C_p\|$. Otherwise, $id_2(\|B_p \text{>} C_p\|, \text{c}) = \varnothing$.
(v) $id_1(\|B_p = C_p\|) = id_1(\|B_p\|) \cup id_1(\|C_p\|)$; $id_2(\|B_p = C_p\|, c) = Val$ when $\|B_p\| = \|C_p\|$. Otherwise, $id_2\|B_p = C_p\|(\text{c}) = \varnothing$.

Definition of truth and validity

A propositional formula A_p of $\mathscr{L}$ is *true* in a possible circumstance c according to a standard model when it is true in that model according to the real assignment $val\mathscr{M}$ of denotations to senses, that is to say iff $val\mathscr{M} \in id_2\|A_p\|(\text{c})$. A propositional formula A_p of $\mathscr{L}$ is *valid* or *logically true* ($\models A_p$) when it is true in all possible circumstances according to all standard models $\mathscr{M}$ of $\mathscr{L}$.

4. A complete axiomatic system

I conjecture that all and only valid formula of $\mathscr{L}$ are provable in the following axiomatic system **MPC**:[27]
The axioms of MPC are all the instances in $\mathscr{L}$ of the following axiom schemas:

Classical truth functional logic

(t1) $(A_p \Rightarrow (B_p \Rightarrow A_p))$,
(t2) $((A_p \Rightarrow (B_p \Rightarrow C_p)) \Rightarrow ((A_p \Rightarrow B_p) \Rightarrow (A_p \Rightarrow C_p)))$
(t3) $((\neg A_p \Rightarrow \neg B) \Rightarrow (B_p \Rightarrow A_p))$

S5 modal logic

(M1) $(\Box A_p \Rightarrow A_p)$
(M2) $(\Box(A_p \Rightarrow B_p) \Rightarrow (\Box A_p \Rightarrow \Box B_p))$

[27]All these axioms are not independent.

(M3) $(\neg\Box A_p \Rightarrow \Box\neg\Box A_p)$

Axioms for tautologies

(T1) (*Tautological* $A_p) \Rightarrow A_p$

(T2) (*Tautological* $A_p) \Rightarrow$ *Tautological Tautological* A_p

(T3) ($\neg$*Tautological* $A_p) \Rightarrow$ *Tautological* $\neg$ *Tautological* A_p

(T4) *Tautological*$(A_p) \Rightarrow ($*Tautological*$(A_p \Rightarrow B_p) \Rightarrow$ *Tautological*$(B_p))$

(T5) *Tautological* $(A_p) \Rightarrow$ *Tautological* $(\Box A_p)$

Axioms for propositional identity (I1) $A_p = A_p$

(I2) $(A_p = B_p) \Rightarrow (C \Rightarrow C^*)$ where C* and C are propositional formulas which differ at most by the fact that an occurrence of B_p replaces an occurrence of A_p

(I3) $(A_p \mapsto B_p \;\&\; (B_p \mapsto A_p)) \Rightarrow (A_p = B_p)$

(I4) $(A_p = B_p) \Rightarrow$ *Tautological* $(A_p = B_p)$

(I5) $\neg(A_p = B_p) \Rightarrow$ *Tautological* $\neg(A_p = B_p)$

Axioms for propositional composition

(C1) $(A_p > B_p) \Rightarrow$ *Tautological* $(A_p > B_p)$

(C2) $\neg(A_p > B_p) \Rightarrow$ *Tautological*$\neg(A_p > B_p)$

(C3) $A_p > A_p$

(C4) $(A_p > B_p) \Rightarrow ((B_p > C_p) \Rightarrow (A_p > C_p))$

(C5) $([(R_n t_1, \ldots, c_n)] > A_p) \Rightarrow (A_p = [(R_n t_1, \ldots, c_n)])$

(C6) $(A_p \wedge B_p) > A_p$

(C7) $(A_p \wedge B_p) > B_p$

(C8) $(C_p > A_p) \Rightarrow ((C_p > B_p) \Rightarrow (C_p > (A_p \wedge B_p)))$

(C9) $A_p \equiv \neg A_p$

(C10) $(\Box[(R_n t_1 \ldots t_n)] > A_p) \Leftrightarrow ((A_p = [(\Box R_n t_1 \ldots t_n)]) \vee (A_p = [(\Diamond R_n t_1 \ldots t_n)]) \vee (A_p = [(R_n t_1 \ldots t_n)]))$

(C11) $\Box\neg A_p \equiv \Box A_p$

((C12) $\Box(A_p \wedge B_p) \equiv (\Box A_p \;\wedge\; \Box B_p)$

(C13) $\Box\Box A_p \;\equiv \Box A_p))$

Axioms for elementary propositions

(E1) $\Box[(R_n t_1 \ldots t_n)] \Leftrightarrow [(\Box R_n t_1 \ldots t_n)])$ And similarly for $\Diamond$.

(E2) $[t = t]$ for any individual term t

(E3) $([t_1 = t_2] \Rightarrow (A_p \Rightarrow A'_p))$ when A'_p differs at most from A_p by the fact that an occurrence of the term t_2 in A'_p replaces an occurrence of the term t_1 which is not under the scope of $\Box$, $>$, ^ or the sign of propositional identity in A_p.

(E4) $^\wedge t_1 = {}^\wedge t_2 \Rightarrow$ *Tautological* $[(t_1 = t_2)]$

(E5) *Tautological* $[(t_1 = t_2)] \Leftrightarrow [(t_2 = t_1)]$

(E6) $^\wedge R_n = {}^\wedge R'_n \Rightarrow$ (*Tautological*$[(R_n t_1 \ldots t_n)] \Leftrightarrow [(R'_n t_1 \ldots t_n)])$

(E7) $((^\wedge t_1 = {}^\wedge d_1) \wedge \ldots \wedge ({}^\wedge t_n = {}^\wedge d_n) \wedge ({}^\wedge R_n = {}^\wedge R'_n)) \Rightarrow ([(R_n t_1 \ldots t_n)] = [(R'_n d_1 \ldots d_n)])$
(E8) $([(R_n t_1 \ldots t_n)] = [(R'_n d_1 \ldots d_n)]) \Rightarrow ({}^\wedge R_n = {}^\wedge R'_n))$
(E9) $([(R_n t_1 \ldots t_n)] = [(R'_n d_1 \ldots d_n)] \Rightarrow (({}^\wedge t_k = {}^\wedge d_1) \vee \ldots \vee ({}^\wedge t_k = {}^\wedge d_n)))$ where $n \geq k \geq 1$
(E10) $\neg({}^\wedge R_n = {}^\wedge R_m)$ when $n \neq m$
(E11) *Tautological* $[(R_2 t_1 t_2)] \Leftrightarrow ({}^\wedge t_1 = {}^\wedge t_2 \wedge (({}^\wedge R_2 = {}^\wedge{=}_2) \vee ({}^\wedge R_2 = {}^\wedge \Box{=}_2)))$
(E12) $\neg$*Tautological* $[(R_n t_1 \ldots t_n)]$ when $n \neq 2$
(E13) $\neg$ *Tautological* $\neg$ $[(R_n t_1 \ldots t_n)]$ when $n \neq 2$

The rules of inference of MPC are:

The rule of Modus Ponens:

(MP) From the sentences $(A \Rightarrow B)$ and A infer B.

The tautologization rule:

(RT) From a theorem A infer *Tautological*A.

5. Valid laws

5.1 Laws about the structure of constituents

A proposition is composed from all the atomic propositions of its arguments. $\models A_p > [(R_n c_1, \ldots, c_n)]$ when $[(R_n c_1, \ldots, c_n)]$ occurs in A_p. Modal propositions have all the atomic propositions of their argument. $\models MA_p > A_p$ where $M = \Box, \Box\neg, \Diamond$ or $\Diamond\neg$ Moreover $\models A_p > [(\Box R_n c_1, \ldots, c_n)]$ when $[(R_n c_1, \ldots, c_n)]$ occurs within the scope of $\Box$ in A_p. So $\Box[(R_n t_1 \ldots t_n)]$ is not an elementary proposition.

All the different modal propositions of the form MA_p have the same structure of constituents.

$\models M\Box A_p \equiv M'A_p$ where M and M′ are $\Box$, $\Box\neg$, $\Diamond$ or $\Diamond\neg$. Thus $\models \Box A_p \equiv \Box\neg A_p$ and $\models \Diamond A_p \equiv \Box A_p$. As one can expect, $\models M(A_p \wedge B_p) \equiv (MA_p \wedge MB_p)$; $\models M(A_p \equiv \Box A_p$ and $\models M\Diamond A_p \equiv \Diamond A_p$
Some modal attributes are identical. $\models {}^\wedge\Box R_n = {}^\wedge\Box\Box R_n$ However, $\not\models {}^\wedge\Box{=}_2 = {}^\wedge{=}_2$.

5.2 Laws for tautologyhood

Tautologyhood is stronger than necessary truth and contradiction stronger than necessary falsehood. $\models$ (*Tautological*A_p) $\Rightarrow \Box A_p$. But $\not\models \Box A_p \Rightarrow$ *Tautological*A_p
There are elementary, modal as well as truth functional tautologies and contradictions.

$\models$ *Tautological* [t = t]; $\models$ ^t$_1$ = ^t$_2$ $\Rightarrow$ *Tautological*[t$_1$ = t$_2$] and $\models$*Tautological* $\Box(A_p \vee \neg A_p)$

5.3 Laws for tautological implication

Tautological implication is much finer than strict implication. $\models$ *Tautological* $(A_p \Rightarrow B_p) \Rightarrow (A_p \multimap B_p)$. But $\not\models (A_p \multimap B_p) \Rightarrow$ *Tautological* $(A_p \Rightarrow B_p)$.

Thus $\models \Box A_p \Rightarrow (B_p \multimap A_p)$. But $\not\models \Box A_p \Rightarrow$ *Tautological*$(B_p \Rightarrow A_p)$. The necessarily true proposition that the biggest whale is a mammal is strictly implied by all propositions. But it is not tautologically implied by any tautology. For it is not tautological. Only tautologies can strongly imply other tautologies.

$\models$((*Tautological* B_p) $\wedge$ *Tautological* $(A_p \Rightarrow B_p)) \Rightarrow$ *Tautological* A_p

Similarly $\models \Box\neg A_p \Rightarrow (A_p \multimap B_p)$. Necessarily false propositions strictly imply all other propositions. But only contradictions can tautologically imply contradictions. $\not\models \Box\neg A_p \Rightarrow$ *Tautological*$(A_p \Rightarrow B_p)$. So only contradictions tautologically imply all other propositions.

All valid laws of material implication of truth functional and S5 modal logic are valid laws of tautological implication. Thus $\models$*Tautological* $(A_p \Rightarrow B_p)$ when $\models(A_p \Rightarrow B_p)$ in S5 modal logic. In particular, $\models$ *Tautological* $(A_p \Rightarrow (A_p \vee B_p))$ and $\models$*Tautological* $(A_p \Rightarrow \Diamond A_p)$. Moreover, $\models$*Tautological* $([\Box R_n c_1 \ldots c_n)] \Leftrightarrow \Box[(R_n c_1 \ldots c_n)])$. And similarly for $\Diamond$. Thus the propositions that John is perturbable and that it is possible that John is perturbed are tautologically equivalent.

Whenever a proposition tautologically implies another, we can have it in mind without having in mind the other. $\not\models$(*Tautological* $(A_p \Rightarrow B_p)) \Rightarrow (A_p > B_p)$) However we could not express both propositions without knowing that the first implies the second. This is why tautological implication generates *weak psychological and illocutionary commitment* in thinking and speaking. Any assertion (or belief) that P *weakly commits* the agent to asserting (or believing) any proposition Q that P tautologically imply.

5.4 Laws for strong implication

Strong implication is the strongest kind of propositional implication. It requires inclusion of content in addition to tautological implication. So there are two reasons why a proposition can fail to imply strongly another. Firstly, the second proposition can require new predications. In that case, one can think the first without thinking the second. $\models \neg(A_p > B_p) \Rightarrow \neg(A_p \mapsto B_p)$. Secondly, the first proposition can fail to imply tautologically the second. In that case, one can ignore even tacitly that it implies the second.

Unlike strict and tautological implications, strong implication is anti-symmetric (Axiom I3). The rule of *Modus Tollens* does not hold for strong implication. $\not\models(A_p \mapsto B_p) \Rightarrow (\neg B_p \mapsto \neg A_p)$

Strong implication is also finer than Parry's analytic implication which is not tautological. $\not\models(A_p \Rightarrow B_p) \Rightarrow (A_p \mapsto B_p)$ For $\not\models(A_p \rightarrow B_p) \Rightarrow$ *Tautological*$(A_p \rightarrow B_p)$.

5.5 Natural deduction

Valid laws of inference of natural deduction whose premises contain the atomic propositions of their conclusion generate strong implication. Thus when $\models(A_p \Rightarrow B_p)$ in S5 modal logic and $\models (A_p > B_p)$ it follows that $\models (A_p \mapsto B_p)$.

This leads to the following system of *natural deduction*:

The law of elimination of conjunction: $\models(A_p \wedge B_p) \mapsto A_p$ and $\models(A_p \wedge B_p) \mapsto B_p$

The law of elimination of disjunction: $\models((A_p \mapsto C_p) \wedge (B_p \mapsto C_p)) \Rightarrow (A_p \vee B_p) \mapsto C_p$

Failure of the law of introduction of disjunction: $\not\models A_p \mapsto (A_p \vee B_p)$. So strong implication is stronger than *entailment* which obeys the law of introduction of disjunction.

The *law of introduction of negation*: $\models A_p \mapsto O_t \Rightarrow (A_p \mapsto \neg A_p)$ where O_t is any contradiction.

Failure of the law of elimination of negation:
$\not\models(A_p \wedge \neg A_p) \mapsto B_p$

Strong implication is paraconsistent.

The *law of elimination of material implication*:
$\models(A_p \wedge (A_p \Rightarrow B_p) \mapsto B_p$

The *law of elimination of necessity*: $\models \Box A_p \mapsto A_p$

The *law of introduction of necessity*: $\models A_p \mapsto B_p \Rightarrow \Box A_p \mapsto \Box B_p$

The *law of elimination of possibility*: $\models \Diamond A_p \mapsto B_p \Rightarrow A_p \mapsto B_p$

Failure of the law of introduction of possibility: $\not\models A_p \mapsto \Diamond A_p$ because $\not\models A_p > \Diamond A_p$

Strong implication is *decidable*.
For $\models A_p > B_p$ when all predication formulas which occur in B_p also occur in A_p. Moreover, $\models$*Tautological* $(A_p \Rightarrow B_p)$ when all the semantic tableaux of S5 modal logic for $(A_p \Rightarrow B_p)$ close.

There is a *theorem of finiteness for strong implication*: Every proposition only strongly implies a finite number of others. In particular, $\models$*Tautological* $B_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p > B_p)$. A proposition strongly implies all and only the tautologies composed from its atomic propositions.

And $\models$*Tautological* $\neg A_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p > B_p)$. A contradiction strongly implies all and only the propositions composed from its atomic propositions.

The decidability and finiteness of strong implication confirm that it is cognitively realized.

5.6 Laws of propositional identity

Modal propositions are richer than modal predications. In particular, $\not\models \Box[(R_n t_1 \ldots t_n)] = \Box[(R_n t_1 \ldots t_n)]$ For $\not\models \Box[(R_n t_1 \ldots t_n)] > \Box[(R_n t_1 \ldots t_n)]$ The failure of such a law is shown in language. Properties such as being the father of a person are possessed by the same male parent in all possible circumstances. These properties have the same extension as their necessitation. But when we think that someone is the father of someone else, we do not *eo ipso* think that he is necessarily his father.

All the classical *Boolean laws of idempotence, commutativity, associativity* and *distributivity* are valid laws of propositional identity:
$\models A_p = A_p \wedge A_p \models (A_p \wedge B_p) = (B_p \wedge A_p) \models (A_p \vee (B_p \vee C_p)) = ((A_p \vee B_p) \vee C_p)) \models \neg(A_p \vee B_p) = (\neg A_p \wedge \neg B_p) \models (A_p \wedge (B_p \vee C_p)) = ((A_p \wedge B_p) \vee (A_p \wedge C_p)) \models \Box(A_p \wedge B_p) = (\Box A_p \wedge \Box B_p)$

So are the laws of *reduction*: $\models \neg\neg\, A_p = A_p \models M\Box A_p = \Box A$ and $\models M\Diamond A_p = \Diamond A_p$ where $M = \Box, \Box\neg, \Diamond$ or $\Diamond\neg$ In particular, $\models \Box A_p = \Box\Box A_p$ and $\models \Box A_p = \Diamond\Box A_p$

Unlike hyperintensional logic, my logic of propositions does not require that identical propositions be *intensionally isomorphic*.[28] Intensional isomorphism is too strong a criterion of propositional identity. However, propositional identity requires more than *co-*

[28]See Max J. Cresswell, "Hyperintensional Logic". *Studia Logica* [1975].

entailment advocated in the logic of relevance. $\not\models A_p \mapsto (A_p \wedge (A_p \vee B_p)$. As M. Dunn pointed out, it is somehow unfortunate that A_p and $(A_p \wedge (A_p \vee B_p)$ co-entail each other.[29] For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new sense.

Finally strong equivalence is finer than *analytic equivalence* $\leftrightarrow$. Consider the following law: $\models [(\Box R_1 c)] \Rightarrow ([(\Box R_1 c)] \leftrightarrow ([(\Box R_1 c)] \vee \neg[(\Box R_1 c)]))$. It is not a valid law of propositional identity.

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[29] See his philosophical rumifications in Anderson *et al* [1992].

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